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Finite Difference Method in Regular Geometry Heat Transfer Problem

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ABSTRACT

Finite difference method (FDM) is a known numerical method for finding approximate solution to boundary value problems (BVP). It is a helpful method in approximating the solutions to differential equation. Numerical methods are very useful in solving engineering problems in many areas related to fluid dynamics, heat and mass transfer problems and other partial differential equations of mathematical physics especially when such problems cannot be solved numerically due to nonlinearities, complex geometries or complicated boundary conditions. However, the application is limited to regular geometry and simple irregular geometry problems.

KEY WORDS: *Finite Difference Method; Heat Transfer; Two Dimensional; Regular Geometry*.

1.0 INTRODUCTION

Finite Difference Method (FDM) is a well-known numerical method to solve problems involving linear second order independent partial differential equations [1]. FDM comprises first discretization of the spatial domain, then the differential equation, the set of boundary conditions, and finally a subsequent solution of a large system of linear equations for the approximate solution values in the nodes of the numerical mesh [2].

Discretization consists of first introducing a mesh of nodes by subdividing the solution domain into a finite number of sub domains [3]. Then approximating the derivatives in the boundary value problem (BVP) by means of appropriate finite difference ratios which can be obtained from a truncated Taylor series. The simplest discretization of operators from second-order differential usually has the accuracy of first or second order [4].

FDM is generally known and acknowledged as the simplest method. The approximations of the derivatives by differences in these values were clarified by substituting the values at a grid points [5]. One of the application of FDM is heat transfer.

Heat transfer is the study of thermal energy transport within a medium of molecular interaction, fluid motion and electro-magnetic waves which resulting from a spatial variation in temperature [6].

The heat conduction problem was presented as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Q=0$$

where T is temperature and Q represents an equation for heat source [7].

Heat equation is one of the most important partial differential equation which describes the distribution of heat or variation in temperature in a given region over time [8]. Basically, the two-dimensional heat equation can be solved theoretically as well as numerically by using numerical method such as the finite difference method.

2.0 PROBLEM STATEMENT

Consider the following steady state heat transfer problem in two-dimensional as shown in Figure 1.

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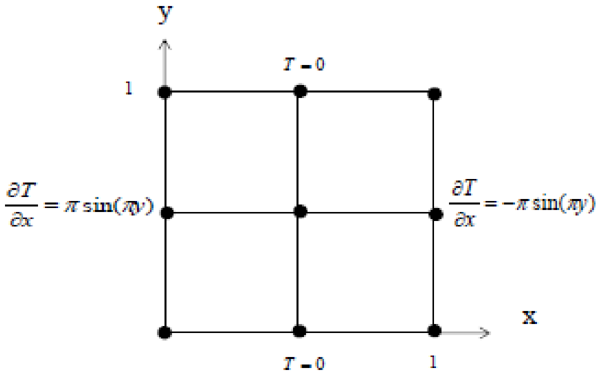

Figure 1: Two-dimensional regular geometry heat transfer problem.

The two- dimensional regular geometry problem is given as follow

$$-\nabla^2 u = f(x,y) \tag{1}$$

and the boundary conditions are

$$u(x,0) = u(x,1) = 0 \tag{2}$$

$$\frac{\partial u}{\partial x}(0,y) = \pi \sin(\pi y) \tag{3}$$

$$\frac{\partial u}{\partial x}(1,y) = -\pi \sin(\pi y) \tag{4}$$

In the equation, u is an unknown scalar function of (*x*, *y*) defined on a square domain [0, 1], and *f* is given function of (*x*, *y*). In this example, the analytical or real solution is given by

$$u(x) = \sin(\pi x)\sin(\pi y) \tag{5}$$

From (1), using (5) we can derive *f* (*x*, *y*). First from the left boundary condition

$$u_x = \pi\cos(\pi x)\sin(\pi y)$$

$$u_{xx} = -\pi^2 \sin(\pi x)\sin(\pi y)$$

$$u_{xxx} = -\pi^2 u$$

then from the right boundary condition

$$u_y = \pi\sin(\pi x)\cos(\pi y)$$

$$u_{yy} = -\pi^2 \sin(\pi x)\sin(\pi y)$$

$$u_{yyy} = -\pi^2 u$$

substitute the derivative back into equation (1)

$$-\nabla^2 u = f(x,y)$$

$$u_{xx} + u_{yy} = -f(x,y)$$

$$-\pi^2 u - \pi^2 u = -f(x,y)$$

$$2\pi^2 u = f(x,y)$$

and finally, we will get

$$f(x,y) = 2\pi^2 \sin(\pi x)\sin(\pi y) \tag{6}$$

Note that the last two natural boundary conditions are the form of (k = 1) here:

$$k\nabla u.n = \pi\sin(\pi y)$$

This is the mathematically precise way of entering natural boundary conditions needed to define the normal flux. The left boundary, n = (-1, 0) and the right boundary n = (-1, 0). Thus, indeed the two natural boundary conditions fit in the above general form.

3.0 METHODOLOGY

Given the mathematical model

$$-\nabla^2 T = f(x,y) \tag{7}$$

equation (7) will equal to

$$\frac{\partial^2}{\partial x^2}T_i^j + \frac{\partial^2}{\partial y^2}T_i^j = f_i^j \tag{8}$$

expand (8) and we will get (9).

$$T_i^j = \frac{\Delta y^2\left(T_{i+1}^j + T_{i-1}^j\right) + \Delta x^2\left(T_{i+1}^j + T_{i-1}^j\right) - \Delta y^2\,\Delta x^2\, f_i^j}{2\,(\Delta y^2 + \Delta x^2)} \tag{9}$$

Then from the left boundary condition

$$u' = \pi\sin(\pi y)$$

$$= f_1(y)$$

we can get the equation to calculate temperature.

$$T' = f_1(y_j)$$

$$\frac{T_{i+1} - T_{i-1}}{2\,\Delta\, x} = f_1(y_j)$$

$$T_{i+1} - T_{i-1} = 2\,\Delta\, x\, f_1(y_j)$$

$$T_{i-1} = T_{i+1} - 2\,\Delta\, x\, f_1(y_j) \tag{10}$$

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Then from the right boundary condition

$$u' = -x\sin(\pi y)$$
$$= f_r(y)$$

we will get another equation to find temperature.

$$T' = f_r(y_j)$$

$$\frac{T_{i+1} - T_{i-1}}{2\,\Delta\,x} = f_r(y_j)$$

$$T_{i+1} - T_{i-1} = 2\,\Delta\,x\,f_r(y_j)$$

$$T_{i+1} = T_{i-1} + 2\,\Delta\,x\,f_r(y_j) \qquad (11)$$

Equations (10) and (11) to calculate the temperature will be the algorithm for MATLAB programming to find the solution for regular geometry heat transfer problem using the application of FDM.

4.0 FINDINGS AND DISCUSSION

Figure 2 illustrate the solution for heat distribution when we applied FDM to solve two-dimensional regular geometry heat transfer problem and we set the step size to equal 100. We can see the highest temperature is 80°C and the lowest is at -80°C with 1000 number of iterations.

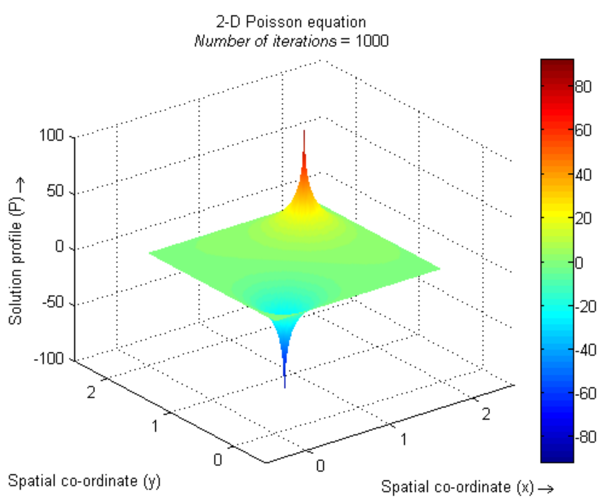

Figure 2: Solution of heat distribution of two-dimensional regular geometry heat transfer problem using FDM when step size = 100.

On the other hand, Figure 3 shows the error of heat distribution. Based on the result we can see that the highest value of error is -0.5 indicated by red region in the graph and the lowest error is -2.5 where in the figure was specified by blue region. The error is very small presenting that the solution is quite truthful to the real solution.

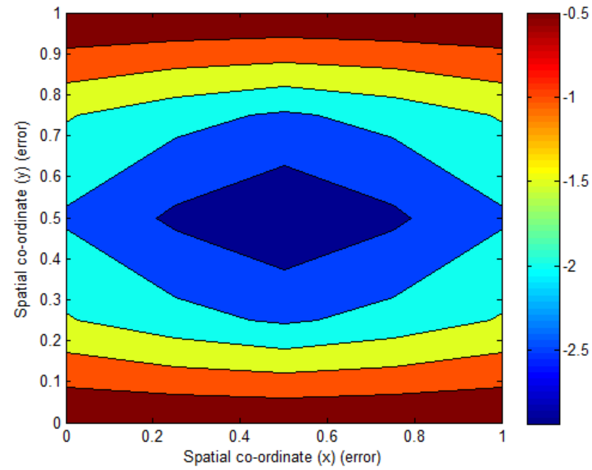

Figure 3: Error of heat distribution of two-dimensional regular geometry heat transfer problem using FDM.

Table I is the obtained temperature when we set the step size to equal two exactly like in the diagram of the problem. We can see that it agrees with the boundary conditions because we set the top and bottom to be equal to zero.

Table 1: Temperature for Solution of Regular Geometry Using FDM.

Nodes	*Temperature / Error (°C)*
1	0
2	0
3	0
4	-0.2247
5	0.1213
6	-0.2247
7	0
8	0
9	0

5.0 CONCLUSION AND RECOMMENDATIONS

FDM can solve regular two-dimensional geometry heat transfer problem. FDM is easier to compute as well as to code compared to other numerical method such as Finite element method. Generally, FDM is a simple method applicable to solve common problems defined on regular geometries or simple irregular geometries.

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