# Energy Transmission to Long Waves Generated by Instantaneous Ground Motion on a Beach

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## ABSTRACT

Historically speaking, all high frequency earth-quakes do not produce tsunami which is evident from several earthquakes that took place in Indian Ocean between 2004 and 2006. The works of Geist et.al ['06] and Ammon et.al ['05] are good examples of such study through which they have pointed out the miserable failures of the existing warning system. The reasons of no-generation of tsunami due to large quakes may be several but in this article we have tried to throw some light on this puzzle and tried to analyze energy transmission to a tsunami motion at steady state and have shown that no energy transmits for certain frequencies of the forcing parameter. We discuss here tsunami waves which are generated by instantaneous bottom dislocation where the ocean floor is taken to be of variable slope and analytical solutions are provided correct to all time t.

**KEY WORDS:** *Tsunami Waves; Shallow Water Equations; Hankel Transform; Hankel Functions; Asymptotic Expansion.*

## NOMENCLATURE

| | |
|---|---|
| $\eta$ | Wave elevation/surface displacement |
| $\eta_{st}$ , $u_{st}$ | surface displacement and velocity at steady state |
| $(J_\nu, Y_\nu)$ | Bessel function of first and second kind of order ν |
| H(t) | Heaviside unit function |
| $H_\nu^{(2)}(z)$ | Hankel function |
| $S_{\mu,\nu}$ | Lommel's function |

## 1.0 INTRODUCTION

Our interest is to study transmission of energy in the generation and propagation of long waves due to underground upheaval in an ocean with variable slope. The approach is an analytical one where we have restricted ourselves in solving forced long linear shallow water equations, the solution of which, it seems, is not found till date in variable ocean floor. For a beach of variable slope $y = -qx^r$ , q > 0, r > 0, referred to horizontal and vertical directions as x-and y-axis respectively waves are generated by an instantaneous ground upheaval, along with a prescribed initial elevation and a velocity of the free surface at the instant before the ground begins to move. In conformity with Tuck and Hwang's analysis of long wave generation due to arbitrary ground motion over a uniformly sloping beach (r = 1), we firstly show that it is possible to find a non-singular solution of the problem for all time t when the ocean slope varies. Then by taking a very general type of time–dependent bottom dislocation we have been able to split the integrals in two parts one representing the waves due to free vibration which we claim to be the forerunners. It is shown that the forced waves (the second contributory part of the wave integral) will eventually catch up these forerunners and occupy the total wave spectrum beyond the half period of the forcing parameter. Assuming a time periodic ground motion, we next show that a steady-state exists. At this stage a noteworthy feature is observed of no transmission of energy from a finitely distributed time-periodic ground motion for a certain set of values of the disturbance function. This kind of paradoxical result was first observed by Stoker for steady-state surface waves in infinitely deep water (Stoker, J.J., 1957). Introduction of small viscosity of the fluid may produce some amount of spreading of energy but that does not explain the huge non-transmission of energy which we found analytically over a large ocean area. Our attempt to find analytical solution of the problem helps us to understand the influence of variable bottom slope on wave elevation and velocity which might be helpful to understand the evolution of tsunami waves induced by near-shore earthquakes [Tinti and Tonini, 2005]. Following the comments of Weahausen and Laitone (Surface waves, 1960), and Pelinovsky (2001) we assert that energy transmission explained here may also prove to be relevant in generation of long waves with variable pressure distributions.

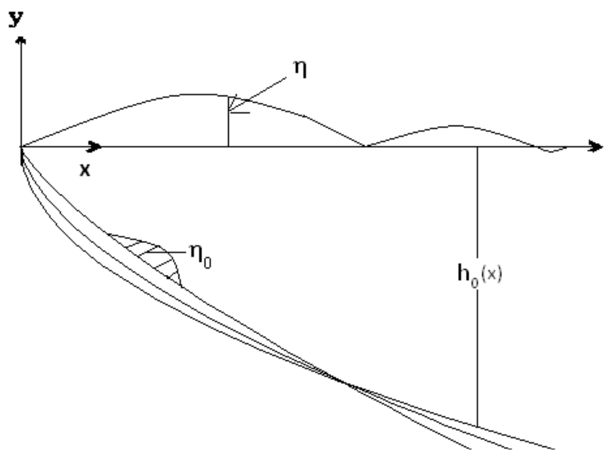



Fig. 1: Schematic diagram of the sea-bottom motion and symbolic definitions.

## 2.0 PROBLEM AND ITS SOLUTION:

We take the vertical upward direction as the y-axis, and the undisturbed horizontal surface of the sea as the xz -plane of which the axis Oz is along the shoreline. The sea is supposed to be bounded by a beach of variable slope given by the equation $y = h_0(x)$ at equilibrium (Fig. 1).

We assume a two-dimensional motion in which long waves are excited by a sudden bottom upheaval of height $\eta_0(x, t)$ accompanied by an initial surface displacement $\eta_1(x)$ together with an initial vertical surface velocity $\eta_2(x)$. If u(x, t) is the horizontal velocity, η(x, t) is the surface displacement and

$$h \equiv h(x,t) = h_0(x) - \eta_0(x,t) \tag{1}$$

is the depth at the point x, at time t > 0, the non-linear shallow water equations are

$$\frac{\partial}{\partial t}(\eta + h) + \frac{\partial}{\partial x}\{u(\eta + h)\} = 0 \quad , \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} = 0 \quad . \tag{3}$$

At t = 0-, we have

$$\eta = \eta_1(x), \qquad \dot{\eta} = \eta_2(x) \qquad \text{on } y = 0 \tag{4}$$

If $\eta$ and $\eta_0$ are small compared to $h_0$ and u is small compared with the local wave speed $\sqrt{gh_0}$, equations (2) and (3), after using (1), may be linearized to

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(uh_0) = \frac{\partial \eta_0}{\partial t}, \tag{5}$$

$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0. \tag{6}$$

Eliminating u(x, t) from (5), (6), and using suffix notation for partial differentiation, we obtain the partial differential equation satisfied by $\eta$:

$$\eta_{tt} - gh_0(x)\eta_{xx} - gh_0'(x)\eta_x = \eta_{0tt} \tag{7}$$

When $h_0$ and $\eta_0$ are given, it is required to determine $\eta$ as the solution of (7) subject to the initial condition (4). The horizontal velocity u is then found from (5); for this purpose, we may impose a physically reasonable boundary condition at x = 0, namely

$$uh_0 \sim h_0\frac{\partial \eta}{\partial x} \to 0 \qquad \text{as } x \to 0 \tag{8}$$

When $h_0(x) = qx^r$, q > 0, r > 0, equation (7) suggests that we consider the solution of the ordinary differential equation

$$\zeta^r v''(\zeta) + r\zeta^{r-1}v'(\zeta) + \beta^2\gamma^2 v(\zeta) = 0 \tag{9}$$

for the determination of $\eta$.

For $\gamma \neq 0$, the general solution of this equation is [Erdélyi et. al. HTF, 1953]

$$v(\zeta) = c_1\zeta^\alpha J_\nu(\beta\zeta^\gamma) + c_2\zeta^\alpha Y_\nu(\beta\zeta^\gamma), \tag{10}$$

where $J_\nu$ and $Y_\nu$ denote respectively Bessel functions of first and second kind of order $\nu$, and

$$\alpha = \frac{1-r}{2}, \quad \gamma = 1 - \frac{r}{2}, \quad \nu = \pm\frac{r-1}{2-r} \tag{11}$$

For $\gamma = 0$, that is r = 2 the general solution of (9) is

$$v(\zeta) = \frac{C}{\zeta} + D, \qquad \text{(C, D) = constants.} \tag{12}$$

Equations (10) and (12) show that $v(\zeta)$ and $v'(\zeta)$ cannot be both finite at $\zeta = 0$ (in other words, $\eta$ and u cannot be both finite at x = 0 unless

$$c_2 \equiv 0, \text{ and } \gamma \neq 0. \tag{13}$$

To fix up the sign in $\nu$ in (11), we consider the two cases:

I. $0 < r < 2$.

Then $\gamma > 0$ and we have, as $\zeta \to 0+$,

$$\eta \sim \zeta^\alpha J_\nu(\beta\zeta^\gamma) \sim \zeta^{\alpha+\nu\gamma} = \zeta^0 \text{ or } \zeta^{1-r},$$

$$u \sim \frac{\partial \eta}{\partial \zeta} \sim 2\zeta \text{ or } \zeta^{-r}$$

according as the sign is + or – in the expression for $\nu$ in (11). Clearly $\eta$ and u are both finite at $\zeta = 0$. If we take the + sign in the expression for $\nu$ in (11), that is if

$$\nu = \frac{r-1}{2-r}.$$

This is also consistent with the condition (8).

II. $r > 2$

Then $\gamma < 0$. As $\zeta \to 0+$ $\zeta^{\gamma} \to \infty$. Then

$$J_\nu(\beta\zeta^{\gamma}) \sim \zeta^{-\gamma/2}$$

$$\eta \sim \zeta^{\alpha - \frac{\gamma}{2}} = \zeta^{-\frac{r}{4}} \to \infty \text{ as } \zeta \to 0+.$$

Therefore the solution is not bounded at the origin when $\gamma < 0$, that is when $r > 2$. Consequently, we confine ourselves to the case $0 < r < 2$ with the value of $\nu$ given by $\nu = \frac{r-1}{2-r}$ in the subsequent part of the article.

To solve the equation (7) subject to the given initial and boundary conditions, we assume that

$$\eta \equiv \eta(x,t) = \int_0^\infty \xi A(\xi,t)\, x^{(1-r)/2} J_\nu(\xi\gamma^{-1}x^{\gamma})d\xi \qquad (14)$$

Using this in (7), we obtain, by means of (9) and (10), with $c_2 \equiv 0$, the integral equation of first kind

$$\ddot{\eta}_0(x,t) = \int_0^\infty (\ddot{A} + \sigma^2 A)\, \xi\, x^{(1-r)/2} J_\nu(\xi\gamma^{-1}x^{r})d\xi \qquad (15)$$

where

$$\sigma = \xi(gq)^{\frac{1}{2}} \qquad (16)$$

Then solution of $\eta$ is obtained with the help of Hankel inversion theorem [Erdélyi et. al., Tables of Int. Trans., (1954)] as

$$\eta(x,t) = \left(\gamma\sqrt{gq}\right)^{-1}\int_0^\infty x^{(1-r)/2} J_\nu(\xi\gamma^{-1}x^{\gamma})d\xi \times$$
$$\times \int_0^\infty \alpha^{(1-r)/2} J_\nu(\xi\gamma^{-1}\alpha^{\gamma})d\alpha \int_0^t \ddot{\zeta}_1(\alpha,s)\sin\sigma(t-s)ds \qquad (17)$$

Where

$$\zeta_1(x,t) = \eta_0(x,t) + \eta_1(x,t)H(t) + t\eta_2(x,t)H(t) \qquad (18)$$

where H is the Heaviside unit function.

We note that for r =1 this expression reduces to that of $\eta$ found for constant slope beach. [Tuck and Hwang, 1972]. To evaluate the above integral we take $\eta_1(x,t) = \eta_2(x,t) = 0$ and $\zeta_1(x,t) = \eta_0(x,t) = \zeta_1(x)T(t)$

where

$$T(t) = H(t-\tau) + (1 - e^{i\omega t})H(\tau - t)\ ,\ \omega = \pi/\tau\ , \qquad (19)$$

we have been able to evaluate the above integral for all time t.

In the above the t-integral reduces to

$$\left(1 + i\omega\int_0^t dt\right)\sum_{m=0}^\infty \frac{1}{2^{m+1}(m+1)!}(\omega t)^{m+2} j_m(\omega t)\sigma\left(1 - \frac{\sigma^2}{\omega^2}\right)^m$$ for $t < \tau$ where

$$j_m(z) = \sqrt{\pi/2z}\, J_{m+1/2}(z)\,,$$

with the help of a very nice result [Erdélyi et. al. HTF, 1953, pp.58]

$$2(\cos t - \cos\theta) = \sqrt{\frac{\pi}{2}}\sum_{m=1}^\infty \frac{(\theta^2 - t^2)2^{1-m}\theta^{\frac{1}{2}-m} J_{m-1/2}(\theta)}{m!}.$$

Then we spilt the $\xi$-integral in two parts one from $\xi$ = 0 to $\xi_0=\omega/\sqrt{(gq)}$ [the first part], and can be evaluated by another result which combines product of two Hankel functions as an integral of a single [Erdélyi et. al., Tables of Int. Trans., 1954, Vol II, pp. 29]

$$J_\nu\left(\xi\gamma^{-1}\alpha^{\gamma}\right)J_\nu\left(\xi\gamma^{-1}x^{\gamma}\right) = \pi^{-1}\left(4\gamma^2\alpha^{\gamma}x^{\nu}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\lambda\xi)^{-2\gamma}J_\nu(\lambda\xi)(\cos\theta)^{2\gamma}d\theta$$

This spilt corresponds to $\eta_{11}$, say, of $\eta$ and corresponds to the free vibration and can be treated as the forerunners. These waves in this spectrum dominate for first few minutes, to be precise for the half period of the quake forcing.

$$\eta_{11} = (2\gamma)^{-2\nu-1}\frac{\xi_0^{2\nu+2}}{\Gamma(\nu+1)}\left\{\left(1 + i\omega\int_0^t dt\right)\sum_{m=0}^\infty \frac{2(\omega t/2)^{m+2}}{(m+1)} j_m(\omega t)\right\}\times$$
$$\times\sum_{n=0}^\infty\left(-\frac{\xi_0^2 x^{2\gamma}}{4\gamma^2}\right)^n \frac{1}{n!\Gamma(n+m+\nu+2)}\times$$
$$\times\int_0^\infty {}_2F_1\left(-n, -\nu-n; \nu+1; \left(\frac{\alpha}{x}\right)^{2\gamma}\right)\zeta_1(\alpha)d\alpha \qquad (20)$$

On the other hand, the second part of $\xi$-integral from $\xi=\xi_0$ to $\infty$ contribute $\eta_{12}$, say, of $\eta$ representing the forced wave part and they catch up the free waves beyond half period $\tau$ and dominate the wave spectrum gradually for $t > \tau$.

## 2.1 Discussion on the Nature of Waves with the Help of Some Illustrative Figures.

Before we proceed further and discuss the steady-state nature of the waves and the energy transmission let us provide some illustrative figures showing the nature of $\eta_{11}$ and $\eta_{12}$ in an attempt to distinguish them foe small time

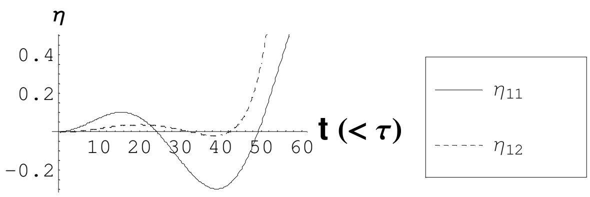



Fig. 2: Depicting $\eta_{11}$ & $\eta_{12}$ for small time when r = 0.7.

The above figure shows the prominence $\eta_{11}$ over $\eta_{12}$ for small time when the value of r = 0.7

The next graph (Fig.2) illustrate nature of $\eta$ for the same value of r = 0.7 and indicates that there might be some sort singularity at t = $\tau$ which needs further analytical investigation for the motion $t > \tau$ for a definite conclusion.

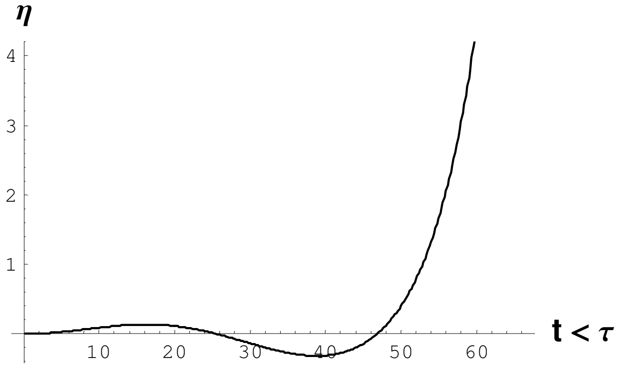



Fig. 3 Depicting $\eta$ for small time when r = 0.7.

We will provide another illustration for another value of r just to show the equivalence of the results for different values of r and the prominence of $\eta_{11}$ over $\eta_{12}$ in small time:

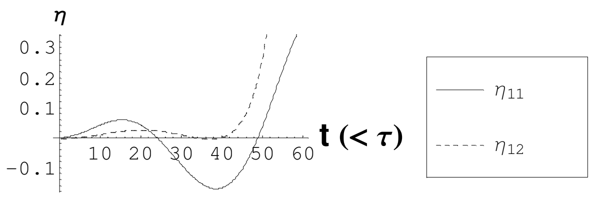
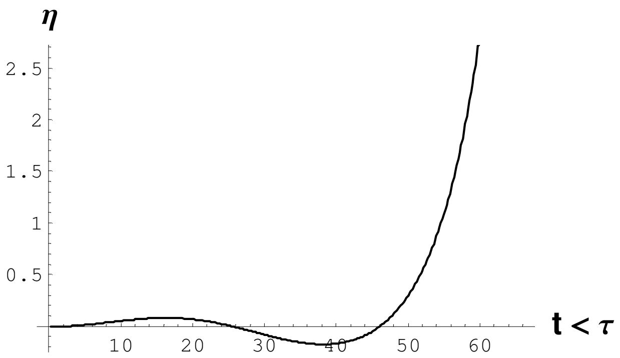



Fig. 4: Depicting $\eta_{11}$ and $\eta_{12}$ for small time when r = 0.8.




Fig, 5: Depicting $\eta$ for small time when r = 0.8.

In the above cases while depicting the results which well established analytically we have taken the half period of the quake forcing that is $\tau$ = 100 seconds and use have been made of the software Mathematica 5.1.

An interesting feature of that comes out of these figure which we have already mentioned that $\eta$ increasing indefinitely indicates as t increasing is probably due to the sudden disappearance of the bottom vibration at t = $\tau$.

In this article although we are interested to discuss the energy transmission at steady-state but it is perhaps not out of context to say few words about $\eta_{12}$, at least qualitatively. The spilt of $\eta$ namely $\eta_{12}$ which comes from the second part of $\xi$-integral in (17) while integrating it from $\xi=\xi_0$ to $\infty$ consists of three parts: one of which has a wave form and the other two are standing disturbances, analytical expressions of which is valid for 2/3 < r < 4/3. We restraint ourselves of writing those complicated expressions rather give some illustration of $\eta_{12}$ below for different sloppiness of ocean floor

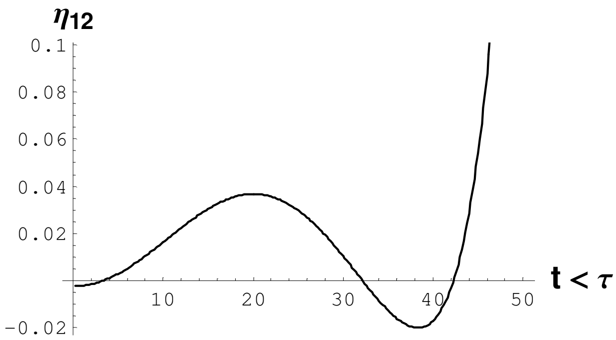



Fig. 6. The graph of $\eta_{12}$ when $t < \tau$ and for r = 0.7

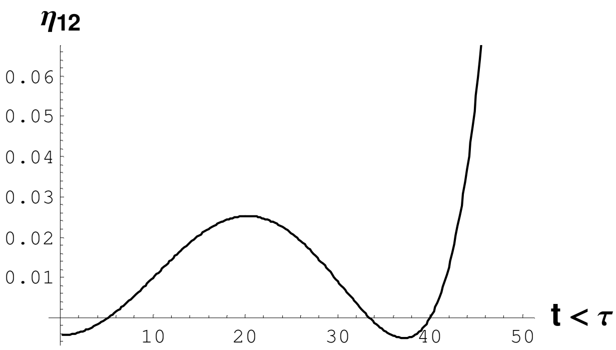



Fig. 7: The graph of $\eta_{12}$ when $t < \tau$ and for r = 0.8.

The figures (7 ) - (8) indicate the dominance of $\eta_{12}$ in the wave spectrum that is going to happen after some time, $\eta_{12}$ actually corresponds to the forced wave part of $\eta$ which will certainly govern the spectrum over those waves which are small and corresponds to the natural frequencies of wave motion.

### 2.2 Periodic Ground Motion: Steady-State Solution Of $\eta$ and u

We assume $\eta_1(x) \equiv 0, \eta_2(x) \equiv 0; \eta_0(x) = f(x)\exp(i\omega t)$, $t > 0$ and show that a steady state ( $t \to \infty$ ) exists and also determine the corresponding values of $\eta$ and u.

### 2.3 Steady-State Values Of η

When the integration with respect to s in (17) is completed, we get

$$\eta = (\omega/\gamma)(gq)^{-1/2} x^{(1-r)/2} \times \left(\frac{\partial}{\partial t} + i\omega\right)$$

$$\int_0^\infty (\sigma - \omega)^{-1} (\omega \sin \sigma t - \sigma \sin \omega t) F(\sigma) d\sigma \qquad (21)$$

where

$$F(\sigma) = (gq)^{-1/2} (\sigma + \omega)^{-1} J_\nu (\xi \gamma^{-1} x^\gamma) \bar{f}(\xi), \qquad (22)$$

$$\bar{f}(\xi) = \int_0^\infty \alpha^{(1-r)/2} J_\nu (\xi \gamma^{-1} \alpha^\gamma) f(\alpha) d\alpha . \qquad (23)$$

We spilt the $\sigma$ -range in (21) into the sub-intervals $[0, \omega/2]$ and $[\omega/2, \infty)$. By the help of known results on Fourier integrals, the part of the integral in (21) over the interval $[0, \omega/2]$ is asymptotically equal to

$$O(t^{-1}) - \sin \omega t \int_0^{\omega/2} ((\sigma - \omega)^{-1} \sigma F(\sigma) d\sigma \text{ as } t \to \infty . \qquad (24)$$

The remaining part of the integral in (21) is written as

$$\eta = \omega \cos \omega t \int_{\omega/2}^\infty (\sigma - \omega)^{-1} F(\sigma) \sin(\sigma - \omega) t d\sigma -$$

$$- \sin \omega t \int_{\omega/2}^\infty (\sigma - \omega)^{-1} \{\sigma - \omega \cos(\sigma - \omega) t\} F(\sigma) d\sigma \qquad (25)$$

Combining (24) and (25), we get for the integral in (21) the expression

$$O(t^{-1}) - \sin \omega t \times (\text{p.v.}) \int_0^\infty (\sigma - \omega)^{-1} \sigma F(\sigma) d\sigma +$$

$$+ \omega \cos \omega t \int_{\omega/2}^\infty (\sigma - \omega)^{-1} F(\sigma) \sin(\sigma - \omega) t d\sigma +$$

$$+ \omega \sin \omega t \times (\text{p.v.}) \int_{\omega/2}^\infty (\sigma - \omega)^{-1} F(\sigma) \cos(\sigma - \omega) t d\sigma$$

$$\text{as } t \to \infty \qquad (26)$$

Here the symbol $(\text{p.v.})\int$ indicates the Cauchy Principal value of the integral in question. Following Bochner [ Wehausen, J.V. and Laitone, E.V. Surface waves. Handbuch der Physik IX (Springer, Berlin, 1960)], the asymptotic values of the third and fourth terms of (26) are respectively

$$\omega \cos \omega t [\pi F(\omega) + O(t^{-1})] \text{ and } O(t^{-1}) \omega \sin \omega t ,$$
$$\text{as } t \to \infty \qquad (27)$$

The results in (27) hold provided

(i) $F(\sigma)$ is differentiable with respect to $\sigma$ in $[0, \infty)$

(ii) $F''(\omega)$ exists,

(iii) $F(\sigma)$ and $F'(\sigma)$ are each absolutely integrable in $[\omega/2, \infty)$.

Equation (21) then gives

$$\eta_{st} = (\pi\omega^2/2\gamma gq) x^{(1-r)/2} \exp\{i(\pi/2 + \omega t)\} \bar{f}(\omega/\sqrt{gq}) \times$$
$$\times [H_\nu^{(2)}(\omega x^\gamma/\gamma\sqrt{gq}) - (2i\nu/\pi) S_{-1,\nu}(\omega x^\gamma/\gamma\sqrt{gq})] -$$
$$-(\omega^2/\gamma gq) x^{(1-r)/2} \exp(i\omega t) \times \int_0^\infty \xi(\xi^2 - p^2)^{-1} \{\bar{f}(\xi) - \bar{f}(p)\} \times J_\nu(\xi\gamma^{-1}x^\gamma) d\xi$$

for $0 < r < 5/3$ (28)

$$h_0 u_{st} = \omega\gamma^{-1} x^{1/2} \exp(i\omega t) \left[ \frac{\pi\omega}{2} (gq)^{-1/2} \bar{f}(p) H_{\nu+1}^{(2)} (p\gamma^{-1} x^\gamma) - \right.$$
$$- ip\nu(\nu + 2) \bar{f}(p) S_{-2,\nu+1} (p\gamma^{-1} x^\gamma) +$$
$$\left. + i \int_0^\infty \xi^2 (\xi^2 - p^2)^{-1} \{\bar{f}(\xi) - \bar{f}(p)\} \times J_{\nu+1}(\xi\gamma^{-1} x^\gamma) d\xi \right]$$

for $0 < r < 5/3$ (29)

Here $Y_\nu$ denotes Bessel function of the second kind, and $S_{\nu,\mu}$ is Lommel's function.

The first term of both $\eta_{st}$ and $u_{st}$, as given below, represent progressive waves:

$$\eta^* = (\pi\omega^2/2\gamma gq) \bar{f}(\omega/\sqrt{gq}) x^{(1-r)/2} \times$$
$$\times \exp\{i(\pi/2 + \omega t)\} H_\nu^{(2)} (\omega x^\gamma/\gamma\sqrt{gq}) \qquad (30)$$

$$h_0 u^* = (\pi\omega^2/2\gamma\sqrt{gq}) x^{1/2} \exp(i\omega t) \bar{f}(p) H_{\nu+1}^{(2)} (p\gamma^{-1} x^\gamma)$$

We also note that $\eta^*$ is an integral of the hyperbolic equation

$$x^r \frac{\partial^2 \eta^*}{\partial x^2} + r x^{r-1} \frac{\partial \eta^*}{\partial x} = (gq)^{-1} \frac{\partial^2 \eta^*}{\partial t^2} \qquad (32)$$

The rest part of $\eta_{st}$ as well as $u_{st}$ represent clearly standing waves. Since $\gamma > 0$ in our case, we may use the asymptotic expansion of $H_\gamma^{(2)}(z)$ for $z \ge 1$ to obtain $\eta^*$ for large x:

$$\eta^* \sim (\pi/2\gamma)^{1/2} (\omega/\sqrt{gq})^{3/2} \bar{f}(\omega/\sqrt{gq}) x^{-r/4} \times$$
$$\times \exp\left[ i\left\{ \omega t + \frac{3\pi}{4} - (\omega x^\gamma/\gamma\sqrt{gq}) + \frac{1}{2}\nu\pi \right\}\right] \qquad (33)$$

The wave described by (33) propagates towards $x \to +\infty$ according to the equation

$$x = (\gamma\sqrt{gq}\, t)^{1/\gamma} \qquad (34)$$

Thus this wave moves with a variable acceleration unless r = 1 when the acceleration is constant [ cp. Tuck & Hwang, 1972, pp - 449]. The height of the wave decreases with the time or distance from the source, according to the factor $x^{-r/4}$ (which is equivalent to $t^{-r/(4-2r)}$ ). Since the depth increases as x, this corresponds to Green's law of shallow water waves.

### 2.4 Transmission of Energy

A notable feature of the steady-state solution is that no energy is transmitted through the liquid for frequencies $\omega = \omega_n$ which make $\bar{f}\left(\omega/\sqrt{gq}\right) = 0$ , and hence $\eta^* = 0$, $u^* = 0$, the part $\eta_{st} - \eta^*$ and $u_{st} - u^*$ being a standing wave. These critical frequencies may form a countable infinite set as it is shown by the following example:

$$\left.\begin{aligned} \eta_0(x,t) &= P\exp(i\omega t), && 0 < x < a \\ &= 0 && x > a \end{aligned}\right\}, t > 0 \qquad (35)$$

with

$$h_0(x) = qx^{1/2} \qquad (36)$$

Then

$$\bar{f}(\xi) = Pa^{5/4}\int_0^a \alpha^{1/4} J_{-1/3}\left(\frac{4}{3}a^{3/4}\xi\alpha^{3/4}\right)d\alpha = \left(Pa^{1/2}/\xi\right)J_{2/3}\left(\frac{a}{3}\xi a^{3/4}\right) \qquad (37)$$

The zeros of $J_\nu(x)$ , for $\nu > -1$ and x real, are known to be countably infinite. If $\gamma_{2/3,n}$ be the n-th position zero of $J_{2/3}(x) = 0$ , the critical frequencies $\omega_n$ are given by

$$\omega_n = \frac{3}{4}a^{-3/4}(gq)^{1/2}\gamma_{2/3,n}, \; n = 1, 2, \ldots \qquad (38)$$

## 3.0 CONCLUSIONS

This study tries to reach out to the big anomaly between the tsunami heights with the so-called early predictions through the energy budget estimation although we know our solution is somewhat crippled as we have restricted ourselves to a linear model. Having said so, we wish to point out that for the study of tsunami wave motion certain important parameters like wave-evolution, the shoaling and wave run-up are well approximated by linear theory, and that too with high degree of precision (Edward Bryant, 2003, Synalokais, CE., 1991). Needless to mention that bathymetric obstacles in large ocean, creating variability of ocean floor starting from continental slope to shoreline plays an important role affecting tsunami translation and the energy transmission not only with teleseismic tsunami but even with tsunamis generated by near-shore earth-quake (Tinti & Tonini, 2005).

Leaving aside the actual physical dislocation of the sea floor the solution provided here is correct for all t. In fact if we apply the sea bed deformation due to earthquake as given by Okada's solution [1992] we may perhaps need to employ some numerical work although in that case one has to remain cautious about oscillatory nature of the wave integrals under consideration. The main purpose of this work is to provide an analytical solution for the waves and discuss qualitatively about those waves for the case when instantaneous motion occurs at sea-bed with variable bathymetry and the energy transmission at the steady state. The place of this bottom dislocation is perfectly arbitrary and is not scaled from the shoreline. This situation is somewhat close to the real tsunami generation mechanism particularly when the non-uniform nature of the slope of ocean comes into play. The effect of the non-uniform nature of the bottom slope is quite visible with the steady-state analysis of the far-field waves. One of the central challenges in tsunami science is to rapidly access a local tsunami severity from the first rough earthquake estimations. In the current state of knowledge, false alarm is perhaps unavoidable (Dutykh et.al. 2012), but the study we proposed here may be taken as a first step to that direction as we have addressed the all important issue of energy transmission to tsunami waves though it is in the steady-state.

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