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A Rainfall Model Comparison by Using Stochastic Neyman-Scott Rectangular Pulse (NSRP) and Bartlett-Lewis Rectangular Pulse (BLRP)

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ABSTRACT

Lately, a climatic change has affected the uncertain occurrence of the rain process that implicates several difficulties in estimating flood disaster. The matter will certainly give the big problem to urban areas. Two stochastic-rain models which use the hourly rainfall data, Neyman-Scott Rectangular Pulse (NSRP) and Bartlett-Lewis Rectangular Pulse (BLRP), are a better way to identify the pattern of the rain events. Five rain characteristics represented by NSRP model's parameter and six characteristics represented by BLRP model's parameter will be used in identifying the rain pattern which is represented by the some rain statistics such as the probability and average of the hourly and daily rainfall. Two statistical rain models will predict the statistical value. This research using the data on 39-year rainfall per hour (1970-2008) from Alorsetar rain station has showed. It has been proved that some statistic models such as the statistical values which are generated by both models are very similar to those of observation statistics or statistics values which are generated from data.

KEY WORDS: *BLRP; Climate change; Flood; NSRP; Urban area*.

1.0 INTRODUCTION

The climate change issue has nowadays resulted in some events related to climate change which has been widely studied by scientists in the world. The torrential rain is one of the components that directly relate to the climate change. Hence, some researches have been studied to explore the characters of the rain deeply, especially for identifying the amount of the rain cell, the heaviness, and the time of the rain event in each storm and some characteristics of the rain such as the rain frequency happened in each one group of rain cell or in other words widely known as 'storm'. These rain characters that are related to cause of flood in a particular region where the frequently occurred rain is followed by the number of the big cell of rain in every storm and the heavy rain in long duration are the cause of flood occurrence in an area.

Two stochastic rain models vizNeyman-Scott Rectangular Pulse (NSRP) and Bartlett-Lewis Rectangular Pulse (BLRP) are the frequently used model resulting in the character of rain. Both the models are very suitable to get complete information about the rain because these models use the rain data in a small scale such as every hour. Rodriguez-Iturbe et al [1],[2] are a scientist that has popularized two stochastic models above. Both of them have been used to the model the rain in Denver city, United Kingdom. Furthermore, other scientists have joined to develop this model vizEntekhabi et al. [3], Cowpertwait et al. [4],[5], Islam et al. [6], and Velghe et al. [7].Two stochastic models have several troubles such as difficulty in finding the rain data to each hour and the complicated mathematics process so that the research in this field is much fewer. In this study, the hourly rain data which are freely available on Alorsetar station in Malaysia Kingdom for 39 years (1970-2008) will be used in generating the stochastic rain models of NSRP and BLRP. The best model will be determined by comparing some statistics that are estimated from both models above to statistics that is produced by the real data. The statistics are the mean of rain per hour, the variance of rain in 1 and 24 hours, and probability of rain in 1 and 24 hours. The model which can estimate and approach the real statistic value is the best model for region around the rain station.

Two stochastic rain models are basically alike because the rain process is calculated in each group or cluster, widely known as storm. The beginning of each storm occurs through the Poisson process where the beginning time mean of storm (λ) is distributed exponentially. In each storm, there is the number of the c rain cell occurring randomly where the rain cells are distributed in a geometry or Poisson having mean (μc).The heaviness and duration of rain occur to every cell in each storm which is distributed exponentially with mean (μx) and (1/η).The difference between the two models lies at the start time of the rain cell (β). The β value in NSRP model is calculated from λ and the β value in BLRP model is calculated to each interval happening between each the rain cell in storm. The description of both rain models can be explained on Figure 1 below.

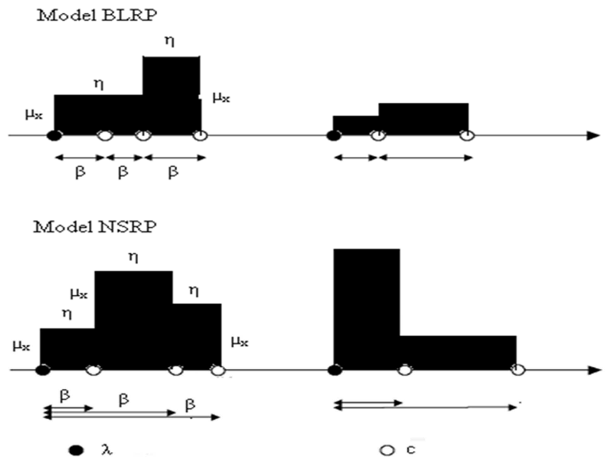

Figure 1: The scheme of the BLRP and NSRP stochastic rain model

2.0 THE MAIN RESULTS

The most important thing in using the rain stochastic models is to estimate the parameter which exists on the models. The BLRP model in this study uses Gamma Distribution which has two parameters namely α and 1/v as the heaviness of rain in every storm. Khaliq and Cunnane [8] have used 6 parameters for this model namely λ, μ_x, α, v, κ, and Φ in which the model used by them exists another parameter such as the duration of the storm distributed exponentially with mean (γ).In this study, two functions, κ=β⁄η and κ=β⁄η, are also given. Cowpertwait et al [4] has used the NSRP model which has 5 parameters namely λ, μ_x, μ_c, β, and η for modeling the stochastic rain in UK. Both of the researches which are done by scientists have given some functions connecting several statistics produced from the observation data such as μ(1) (average rain 1 hour), σ(1), σ(6), σ(24) (the rain variation 1, 2, and 24 hours), ρ(1,1), ρ(1,24) (auto correlation l at 1 for I and 24 hours), φ(1) and φ(24) (the rain probability 1 and 24 hours). The appendix A is given for explaining correlation between the function of statistics and the NSRP parameter. Both of them use numerical way of getting the parameter value of the stochastic rain model.

Table 1: The parameter of BLRP model

Month	Λ	μ_x	A	κ	ϕ	ν
Jan	0.006	71.7	8.8	0.04	0.008	0.114
Feb	0.009	13.6	16.7	0.16	0.073	3.911
March	0.014	2.1	3.4	17.21	0.009	0.005
Apr	0.023	11.9	30.5	0.06	0.063	12.073
May	0.025	9.9	164.5	0.09	0.069	76.732
Jun	0.020	10.6	9.1	0.15	0.062	2.985
Jul	0.024	8.9	5.4	0.14	0.068	1.872
Aug	0.025	11.5	3.9	0.16	0.050	0.853
Sep	0.038	10.5	5.5	0.13	0.050	1.217
Oct	0.038	3.9	3.8×10^{15}	0.02	0.146	8.01×10^{15}
Nov	0.031	11.3	8.4	0.13	0.064	2.076
Dec	0.013	6.0	8.9	0.13	0.059	3.522

Table 2: The parameter of NSRP model

Month	λ	μ_x	μ_c	β	H
Jan	0.005	8.84	2.53	0.23	3.29
Feb	0.007	12.50	2.99	0.21	2.89
March	0.013	11.28	3.10	0.28	2.92
Apr	0.021	11.70	2.20	0.13	2.27
May	0.021	9.93	2.64	0.14	2.05
Jun	0.015	10.02	3.69	0.11	2.08
Jul	0.017	8.29	3.40	0.09	1.66
Aug	0.016	9.75	4.49	0.08	1.87
Sep	0.027	9.13	3.79	0.11	2.29
Oct	0.029	8.96	3.03	0.11	2.02
Nov	0.025	10.17	2.97	0.15	2.53
Dec	0.009	5.78	3.87	0.09	1.81

Table 3: Observation Statistic of the rain data in Alorsetar station

Mon	μ(1)	σ(1)	σ(6)	σ(24)	ρ 1,1
Jan	0.04	0.29	2.97	15.18	0.24
Feb	0.09	1.38	14.69	77.84	0.28
Mar	0.14	2.08	19.99	95.98	0.29
Apr	0.22	3.42	32.22	139.37	0.32
May	0.25	3.61	38.45	162.89	0.36
Jun	0.24	3.08	39.26	182.86	0.36
Jul	0.27	3.21	39.13	206.02	0.41
Aug	0.34	4.78	57.14	343.51	0.39
Sep	0.38	4.42	48.29	255.08	0.33
Oct	0.35	4.22	51.00	237.36	0.35
Nov	0.27	3.43	37.67	167.43	0.30
Dec	0.11	0.86	10.19	55.17	0.40

In this study, the result of parameters of the BLRP and NSRP rain model per month can be seen on Table 1 and Table 2. The parameters are generated after needed statistic value is obtained. This statistic value is observation statistic obtained from the real rain data.

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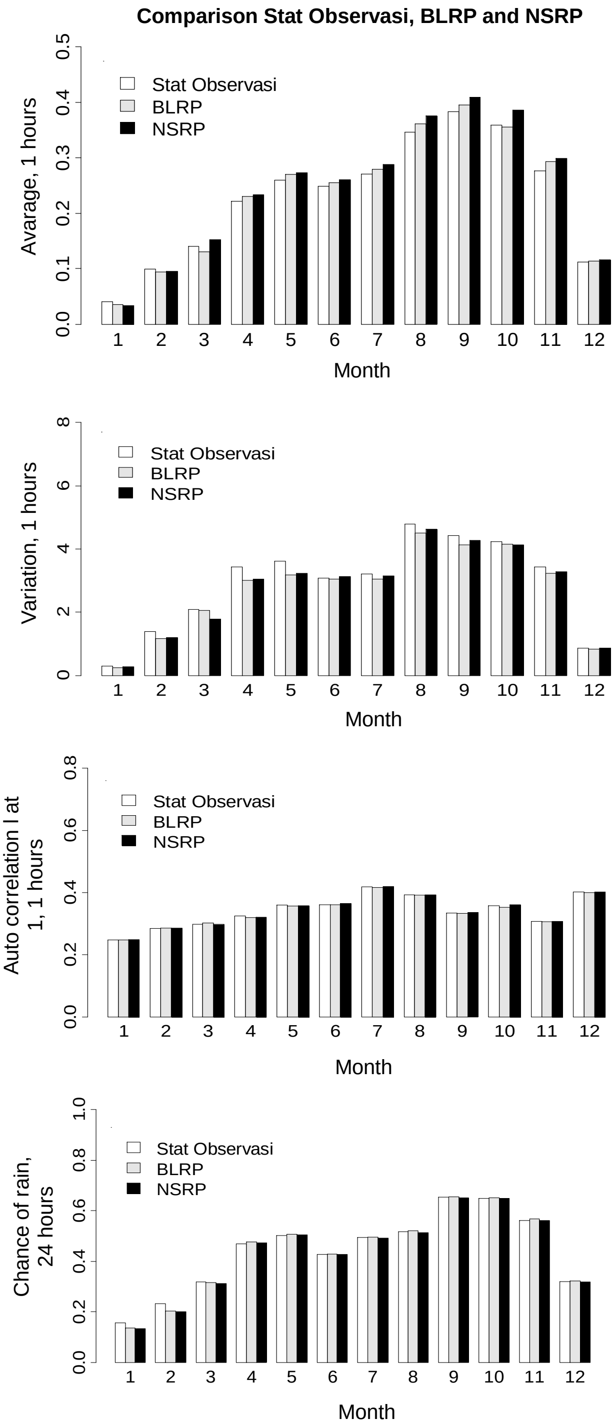

Figure 2: Comparing model and observation statistic

The result of both the rain models is done by comparing some of the statistic models vizμ(1), σ(1), ρ(1,1), and φ(24) which are produced by both models for statistic observation. Based on the Figureureures, it can be concluded that both of the rain models have successfully been used in predicting some statistics which are used for representing the characteristic of rain well, and it is proved by the ability of statistics to approach the statistic of observation.

4.0 CONCLUSION

The modeling of rain using short-scale rain data as hourly rain data is very usefull to produce the complete information about behavior of rain.The two stochastic rain models such as BLRP and NSRP have often been used for this objective, besides, theses rain models are also suitable to predict some statistics which is very important to represent the rain condition in certain areas. To predict the important statistic values for several objectives, the stochastic rain model of BLRP has the same a good capability as NSRP.

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Appendix A

The relationship between observation statistic and NSRP parameter
The equalities (1), (2), (3), and (4) are correlation among mean, variance, autocorrelation, and probability of the rain observation to NSRP parameter.

$$E\left(Y_i^{(\tau)}\right) = \frac{\lambda}{\eta} E(C)E(x)\tau \tag{1}$$

$$Var\left(Y_i^{(\tau)}\right) = \Omega_1\big(\lambda, E(C), E(x)\big)\Psi_1(\eta,\tau) + \Omega_2\big(\lambda, E(C), E(x)\big)\Psi_2(\beta,\eta,\tau) \tag{2}$$

$$Cov\left(Y_i^{(\tau)}, Y_{i+k}^{(\tau)}\right) = \Omega_1\big(\lambda, E(C), E(x)\big)\Psi_3(\beta,\eta,\tau) + \Omega_2\big(\lambda, E(C), E(x)\big)\Psi_4(\beta,\eta,\tau) \tag{3}$$

$$1 - Pr\{Y_i^{(\tau)} = 0\} \tag{4}$$

with,

$$Pr\left\{Y_i^{(\tau)} = 0\right\} = exp\left(-\lambda\tau + \lambda\beta^{-1}(E(C)-1)^{-1}\omega - \lambda\int_0^\infty [1 - p(t,\tau)]dt\right)$$

$$p(t,\tau) = (\,exp[-\beta(t-\tau)] + 1 - \vartheta) \times exp\left(-\frac{(E(C)-1)\beta\upsilon}{\nu} + (E(C)-1)\,exp[-\beta(t+\tau)]\right)$$

$$\Omega_1\big(\lambda, E(C), E(x)\big) = 2\lambda E(C)E(x^2)$$

$$\Omega_2\big(\lambda, E(C), E(x)\big) = \lambda E(C^2 - C)E^2(x)$$

$$\Psi_1(\eta,\tau) = \frac{1}{\eta^3}(\eta\tau - 1 + exp(-\eta\tau))$$

$$\Psi_2(\beta,\eta,\tau) = \Psi_1(\eta,\tau)\frac{\beta^2}{\beta^2-\eta^2} - \frac{\beta\tau - 1 + exp(-\beta\tau)}{\beta(\beta^2-\eta^2)}$$

$$\Psi_2(\beta,\eta,\tau) = \frac{1}{2\eta^3}(1 - exp(-\eta\tau))^2 exp(-\eta(k-1)\tau)$$

$$\Psi_4(\beta,\eta,\tau) = \Psi_3(\beta,\eta,\tau)\frac{\beta^2}{\beta^2-\eta^2} - \frac{(1 + exp(-\beta\tau)^2 exp(-\beta(k-1))}{2\beta(\beta^2-\eta^2)}$$

$k = auto\ correlation$
$\tau = scale\ of\ rain$

$$\omega = 1 - exp[1 - E(C) + (E(C)-1)\,exp(-\beta\tau)]$$

$$\vartheta = \frac{[\eta\, exp(-\beta t) - \beta\, exp(-\eta t)]}{[\eta - \beta]}$$

$$\upsilon = [exp(-\beta t) - exp(-\eta t)]$$

$$\nu = [\eta - \beta] - (E(C)-1)\,exp(-\beta t)]$$

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