Journal of Ocean, Mechanical and Aerospace
-Science and Engineering-, Vol. 15

January 20, 2015

Wake Oscillator Model for Vortex-Induced Vibrations Predictions on Low Aspect Ratio Structures

Mohd Asamudin A Rahman,[a,b*] Krish Thiagarajan,[c] Jeremy Leggoe,[a] and Ahmad Fitriadhy,[b]

a)*School of Mechanical Engineering, University of Western Australia, Crawley, WA, Australia*
b)*School of Ocean Engineering, Universiti Malaysia Terengganu, Terengganu, Malaysia*
c)*Department of Mechanical Engineering, University of Maine, Orono, USA*

*Corresponding author: mohdasamudin@umt.edu.my

Paper History

Received: 29-December-2014
Received in revised form: 14-January-2015
Accepted: 19-January-2015

ABSTRACT

A phenomenological Wake Oscillator Model (WOM) is studied to capture the coupling effects between the fluid and the structure. The Vortex-Induced Vibration (VIV) phenomenon is modelled to describe the motion imposed by the lift forces on the structure. The influence of the aspect ratio (*L/D*) was introduced into the model to characterize the VIV phenomenon for finite cylinders. The proposed model captured the basic features of the VIV such as the amplitude of vibration, frequency, and lift coefficient by coupling the structural equation to the wake equation. Predictions of the WOM are discussed and compared with the experimental data in order to establish a relationship describing VIV as a factor of aspect ratios.

KEY WORDS: *Vortex-Induced Vibrations; Wake Oscillator Model, circular cylindrical structure; aspect ratio effects*

1.0 WAKE OSCILLATOR MODEL

Offshore structures such as jacket platform, risers, mooring lines, Spars, and pipelines, are subject to severe climate and ocean conditions. These structures undergo unremitting forces from the current or wave which resulting in fatigue due to vibration. One of the identified problem is due to Vortex-Induced Vibrations (VIV). VIV is a phenomenon in a fluid flow caused by the shedding of vortices behind the structures due to the interactions of fluid and structure. Comprehensive review have been done by King [1], Sarpkaya [2], Bearman [3], Pantazopolous [4], Williamson and Govardhan [5], and books of Chen [6], Blevins [7], and Sumer and Fredsoe [8], to name a few.

Studies of the VIV have been carried out using different approaches such as experiments, numerical and analytical study. Each one of the approach contribute significantly to the expansion of the knowledge in prediction of VIV. In the present study, the main interest is on Wake Oscillator Model (WOM), as a semi-empirical model for a prediction of the VIV phenomenon in a fluid flow. There are three main types of semi-empirical model, as summarized by Gabbai and Benaroya [9] which are, Wake-body coupled models or wake oscillator model (WOM), Single degree of freedom models (SDOF), and Force decomposition models.

Bishop and Hassan [10], Hartlen and Currie [11], Skop and Griffin [12], Iwan and Blevins [13], Landl [14], Griffin [15], Facchinetti et al. [16] are among the contributors to the development of WOM. Meanwhile, Simiu and Scanlan [17], Goswami et al. [18] use SDOF models to describe the behaviour of the structural oscillator. Sarpkaya [19] is credited for the force decomposition models and succeeded later by Griffin and Koopman [20].

The prediction models are based on the equation of motion of a flexible mounted structure in the transverse direction with 2D flow as defined by [8]. The equation for a single degree of freedom system can be generally written as:

Mass term + Damping term + Stiffness term = Forcing term

$$m\ddot{y} + c\dot{y} + ky = F \qquad (1)$$

where m is the total mass of the system, c is the structural damping, k is spring stiffness, y is the cross-flow displacement and F is the forcing term. A dot over the symbols denotes differentiation with respect to time. This equation is known as structure oscillator. Figure 1 shows a schematic diagram for an elastically supported rigid cylinder.

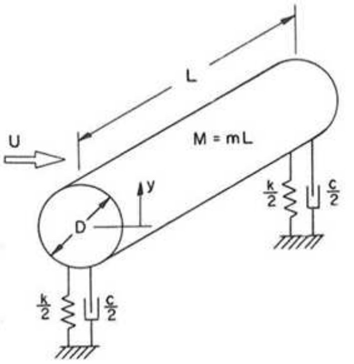

Figure 1: Elastically supported rigid cylinder [13].

Birkhoff and Zarantanello [21] first introduced a model called wake oscillator in order to determine the vortex shedding frequency by theoretical formula. Following this pioneering contribution, numerous works have proposed solving VIV problems by analytical approaches. Bishop and Hassan [10] suggested representing the time-varying forces on a cylinder due to vortex shedding by using a van der Pol type oscillator. Hartlen and Currie [11] proposed a lift oscillator model by coupling the lift force to the cylinder motion (equations (2) and (3)) that satisfies the following characteristics:

i. The oscillator should be self-exciting and self-limiting which means the lift force in the lock-in region is periodic and infinite.
ii. The natural frequency of the oscillator should be proportional to the uniform flow velocity and coincide with the vortex shedding frequency.
iii. The cylinder motion and the oscillator must be connected dynamically.

$$\ddot{y}_r + 2\zeta\dot{y}_r + y_r = a\omega_o c_L \tag{2}$$

$$\ddot{c}_L - \alpha\omega_o\dot{c}_L + \frac{\gamma}{\omega_o}(\dot{c}_L)^3 + \omega_o^2 c_L = b\dot{y}_r \tag{3}$$

where y_r is the dimensionless amplitude, c_L is the lift coefficient, ξ is the damping factor, ω_o the frequency ratio, and a, α, b, γ are parameters defined to best fit the equation to the experimental data. α and γ are the linear and nonlinear term for damping. Both equations were made dimensionless by the following definition;

i. Dimensionless displacement

$$y_r = \frac{y}{D} \tag{4}$$

ii. Dimensionless time

$$\tau = t\sqrt{\frac{k}{m}} = \omega_n t \tag{5}$$

iii. Damping ratio

$$\xi = \frac{c}{2m\omega_n} \tag{6}$$

iv. Dimensionless parameter

$$a = \frac{\rho D^2 L}{8\pi^2 S_t^2 m} \tag{7}$$

v. Frequency ratio

$$\omega_o = \frac{f_s}{f_n} = S_t\left(\frac{U}{f_n D}\right) \tag{8}$$

The model captures most of the features of VIV, such as amplitude response, frequency ratio and hysteresis as shown in figure 2, where stable branches of the periodic motions are represented by solid lines, while unstable branches are represented by dotted lines. Hysteresis phenomenon is indicated by arrows on the plots.

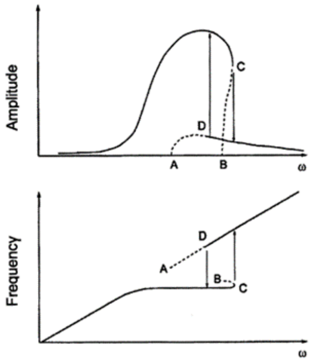

Figure 2: Amplitude and frequency response from Hartlen and Currie model [9]

Facchinetti et al. [16] examine three different coupling terms, namely displacement coupling, $f = Ay$, velocity coupling, $f = A\dot{y}$ and acceleration coupling, $f = A\ddot{y}$, where f is defined as the forcing term, to be coupled with the wake equation. The nonlinear fluctuating lift equation or wake oscillator defined by Nayfeh [22] can be written as:

$$\ddot{q} + \varepsilon\Omega_f(q^2 - 1)\dot{q} + \Omega_f q = F \tag{9}$$

where q is the dimensionless wake variable, Ω_f is the angular frequency, ε is the structure reduced damping and F is the forcing term. The wake oscillator was coupled with the structure oscillator equation by the introduction of dimensionless time and space. It was found that acceleration coupling succeeds best in predicting the VIV response. The stable solution of the acceleration coupling covered most the data collected from the experiments. Meanwhile, the velocity and displacement coupling fails to predict the phase compared to the experimental data.

Due to the complexity of vortex shedding in the near wake region, an exact solution for the fluid structure interaction is yet to be found. Therefore, an approximate model must be constructed to predict the physics of the VIV [13]. However, in most of these wake oscillator models, the 3D effects were not considered [9]. Hence, in this paper, the reliability of the van der Pol equation to capture the 2D phenomenon has been extended to a 3D model with the influence of aspect ratio included for a better prediction of the dynamic response of structures to VIV. To the authors' knowledge, there are only few studies on effects of *L/D* have been conducted in water channel for low cylinder aspect ratio (e.g., $L/D < 10$). For some latest work along this line, see Rahman et al. [23] and Gonçalves et al. [24]. The main objectives in developing this phenomenological model are to validate our VIV experimental data from [23] and to capture any complex behaviour of the VIV phenomenon observed in experiments.

In the present study, a coupled WOM from the literature have been examined to predict the behaviour of the VIV phenomenon and compared with the experimental results. For all aspect ratios, the WOM was solved using the parameters in table 1. Meanwhile, the value of C_D and C_L over U_r was extracted from the experimental results by Rahman et al. [23].

Table 1: Parameters for aspect ratio examined. (Extracted from Rahman et al. [23])

Symbol	*L/D*	*L*	*D*	*k*	*m*	f_n	ζ	*St*
Exp. 1	13	0.78	0.06	245	5.734	0.99	0.031	0.1455
Exp. 2	10	0.6	0.06	245	4.411	1.12	0.031	0.1292
Exp. 3	7.5	0.6	0.08	245	7.841	0.865	0.036	0.1105
Exp. 4	5	0.4	0.08	245	5.228	1.035	0.032	0.0983
Exp. 5	3	0.33	0.11	52	8.154	0.508	0.026	0.1111
Exp. 6	2	0.22	0.11	52	5.436	0.569	0.021	0.1003
Exp. 7	1	0.16	0.16	52	5.926	0.543	0.030	0.0910
Exp. 8	0.5	0.08	0.16	52	4.182	0.695	0.034	0.0670

2.0 Modified Wake Oscillator Model

Coupled WOM by Facchinetti et al. [16] was introduced and modified to suit the flow problem, in order to capture the behaviour of the VIV based on the present experimental data. The coupled model considered the aspect ratio effects in the model with the inclusion of a few assumptions and parameters. It should be emphasized that the development of the WOM will not capture the whole behaviour of the VIV. However, the present attempts are to model the VIV phenomenon based on the data obtained in our experiments. Additionally, the attempts are also to interpret some of the behaviour observed in the experiments with the various parameters included in the present WOM.

Facchinetti et al. [16] discussed the importance of coupling effects on the wake oscillator model performance. This provided significant improvements in the WOM from the classical model introduced by Hartlen and Currie [11] and other researchers before. Coupling equations of structure and wake flow from Facchineti et al. [16] were used to investigate the effects of aspect ratio in the fluid-structure interaction model. The selection of the acceleration coupling term was based on the conclusion given by Facchinetti et al.[16], who succeeded to model the lock in domain for single degree of freedom case. The structure and wake oscillators equations defined by Facchinetti et al. [16] were;

$$\ddot{y} + \left(2\xi\delta + \frac{\gamma}{m*}\right)\dot{y} + \delta^2 y = s \tag{10}$$

$$\ddot{q} + \varepsilon(q^2 - 1)\dot{q} + q = A\ddot{y} \tag{11}$$

where ξ is the structure reduced damping, δ is the reduced angular frequency of the structure, γ is the stall parameter, $m*$ is the mass ratio, and ε and A are tuning parameters.

The dimensionless fluctuating lift force term, s on the RHS of equation (10) is defined as

$$s = Mq \tag{12}$$

$$M = \frac{C_{Lo}}{2}\frac{1}{8\pi^2 St^2 m*} \tag{13}$$

where C_{Lo} is the reference lift coefficient, M is the mass number which scales the effects of the wake on the structure, and q is the fluid variables (refer Fachhinetti et al. [16] for details). In this coupled term, Facchinetti et al.[16] recommended a value of $C_{Lo} = 0.3$ for a large range of Reynolds number ($300 < \mathrm{Re} < 1.5 \times 10^5$). In an attempt to solve the coupled equations for an oscillating cylinder, the value of $C_L o$ is chosen as the lift values derived from free oscillating cylinder experimental results instead of reference values for stationary cylinder, as suggested by [16]. The same consideration was applied to the calculation of the stall parameter. The stall parameter, γ is related to the mean sectional drag coefficient of the structure and given by

$$\gamma = \frac{C_D}{4\pi St}, \tag{14}$$

Facchinetti et al. [16] suggested amplified drag coefficient, $C_D = 0.2$ to simplify the model, based on the motion in the transverse direction. However, to improve the validity of the model in comparison with the experimental data, the value of C_D in the equation was defined as the mean drag coefficient value acquired from single degree of freedom free oscillating experiments presented in Rahman et al. [23]. Both C_{Lo} and C_D were varied for each reduced velocity examined. That means the behaviour of the wake can be specifically captured at each Reynolds number by solving the coupled equations.

The coupled system equation was solved using numerical techniques in MATLAB. The parameters used to solve the coupled equations (10) and (11) were derived from experimental results for different *L/D* by [23]. The equations were solved to capture the structural responses for the aspect ratio investigated. The value of the van der Pol parameter, ε and the scaling of coupling force, A were varied to obtain a best fit with the experimental plot.

3.0 RESULTS AND DISCUSSIONS

Figure 3 shows a series of the results obtained from solving the coupled equations. Based on the results acquired, it is seen that there was a good agreement between the present coupled models and the experimental values. The WOM captured the response amplitudes of the structure over reduced velocities investigated in the experiments, and established the best fit for varying values of ε and A in each case.

The results comparing the model with the experimental values such as plotted in figure 3. Experiment 8 (L/D = 0.5) in figure 3 (h) shows a higher value of the amplitude response at higher reduced velocities. This is possibly due to three-dimensional effects captured by the present WOM. Note that the Strouhal values used in the present WOM development were derived from the experiments and significantly lower from the published value of St = 0.2.

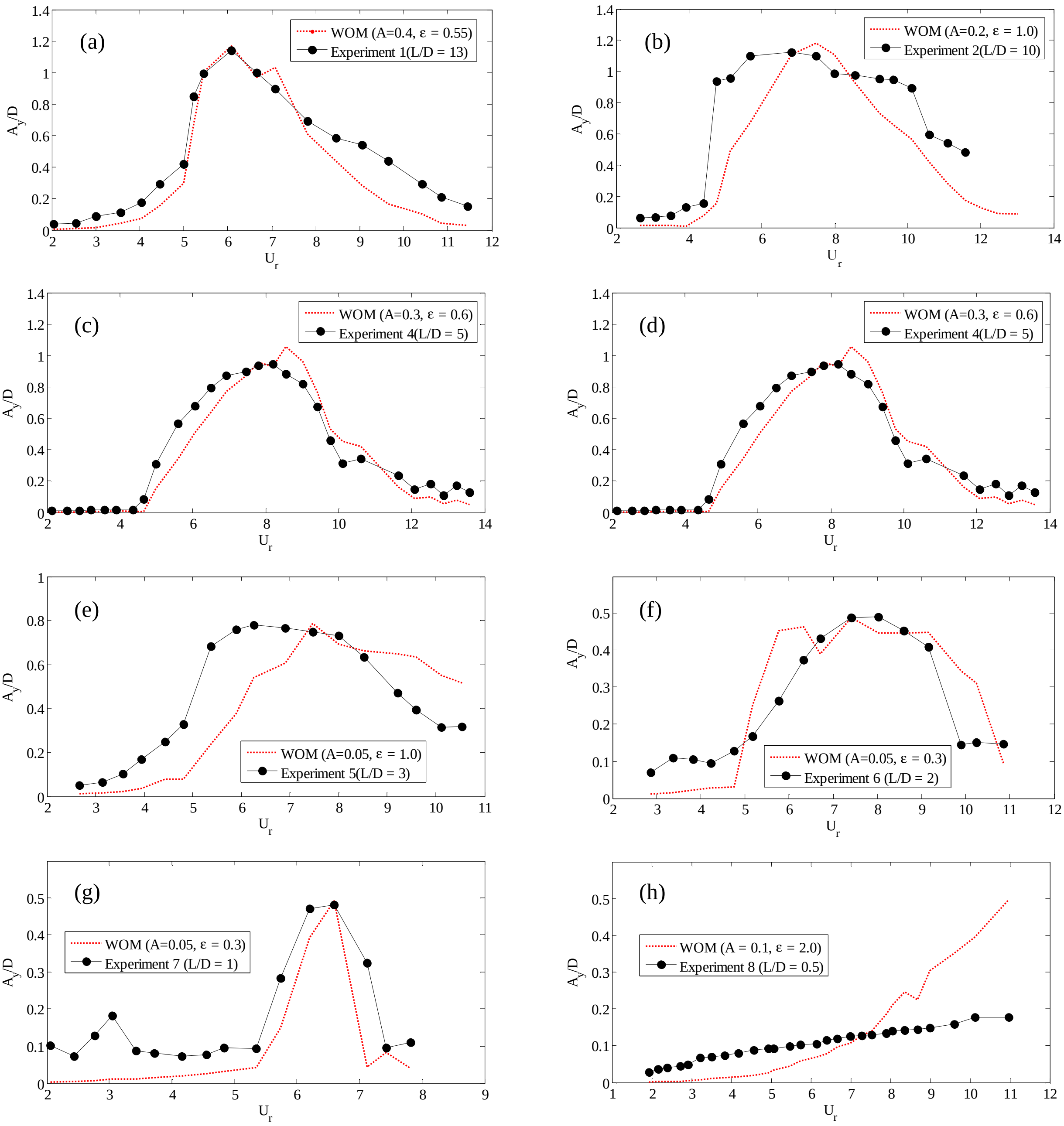

Figure 3: (a) – (h). Comparisons of present coupled WOM with experimental data for each aspect ratio investigated.

These lower values of *St* corresponded to the behaviour of the lock-in region and three-dimensional effects of the end condition. However, there are similarities in the curve shape and pattern in most of the aspect ratios compared to the experiments. The model has been tuned to find the best values in order to capture the stable solutions for the present WOM model. Table 2 shows the value of model parameters corresponding to each aspect ratio plotted in figure 3 (a) – (h). Although the present coupled model shows a fairly good agreement with our experimental data, the suitability of the model is highly dependent on the parameters used. It should be noted that in the present study, no effort has been made to validate the model with other experimental data from other literature.

Table 2: Model parameters value for each *L/D*.

L/D	13	10	7.5	5	3	2	1	0.5
A	0.4	0.2	0.5	0.3	0.2	0.05	0.05	0.1
ε	0.55	1.0	0.5	0.6	0.8	0.2	0.3	2

Good agreement was observed for all aspect ratios investigated which validated the present coupled model for the VIV prediction, except for *L/D* = 0.5 (figure 3 a-h). The success of the present WOM to capture most of the behaviour of the response amplitude in VIV phenomenon solely depends on the model parameters. This is supported by observations from experiments and WOM where at *L/D* =13 there is remarkable agreement whereas at *L/D* = 0.5, VIV was completely disturbed by the flow. These observations were discussed earlier, where the aspect ratio influenced the disturbance in the flow. This is supported by observation in the experiments, where free surface and end conditions were more dominant in the case of low aspect ratio compared to high aspect ratio.

The values of lift and drag coefficients, Strouhal number, damping ratio, and a mass ratio from the experiments were used in the present WOM. The model also depends on the tuning parameters used. As mentioned before, the van der Pol parameter, *ε* and the scaling of coupling force *A* were varied to obtain the best fit to the experimental plot. A relationship of the tuning parameters can be drawn in order to capture the dependency of the model on the tuning parameters. Figure 4 shows the plot for the model parameter ratio, *A/ε* over the aspect ratio tested. It appears that the values of *A/ε* decreased when the aspect ratio decreases for *L/D* = 7.5 to 0.5. However, *L/D* = 13 and 10 yield a slightly lower value compared to *L/D* = 7.5, apparently limiting the trend. This result for *L/D* = 7.5 to 0.5 shows an interesting relationship between the coupling equations and the aspect ratio.

The scaling of the coupling force, *A* was proportional to the response amplitude of the structure in the model. The response amplitude was shown in previous chapters to be decreased as the aspect ratio reduced and suggest that the coupling force was reduced correspondingly. However, the relationship between the model parameter and the aspect ratio remains unclear as the results obtained for *L/D* =13 and 10 differ from the expected relationship. This is possibly due to scaling force coupling parameter, *A* which is lower than that used for *L/D* = 10 as shown in table 2. Figure 5 shows the plots for the Strouhal Number values over model parameter ratio. It can be seen that there is no definite relationship demonstrated by the plot. This is possibly due to the model parameter being fitted in the equations in order to capture the VIV response, which could be an imperfection of the WOM presented. These results should support the observations made for the Strouhal number in experiments for the VIV responses with different aspect ratio.

It can be concluded that for high aspect ratios (*L/D* = 5 – 13), VIV is well established and the WOM models provide sound predictions of the VIV behaviour, as evidenced by the findings of this study. Meanwhile, for low aspect ratios, the VIV phenomenon competes with both free surface and end effects. As the aspect ratio decreases, the WOM model becomes less relevant, as VIV is less established and the free surface effects diminish the accuracy of the model. Thus, for high aspect ratios of 5 and above, the WOM approaches may be used to predict VIV on floating structures. However, for aspect ratios less than 5, experimental scale model are needed to predict the extent (if any) of VIV behaviour. WOM come into the picture when the fundamental understanding of VIV required in the investigation, and considerably satisfied with the experimental data and completing the VIV investigation with further understanding of VIV phenomenon. These significant results in this study have been produced by solving the wake and structure equation, which extend the available analytical model for low aspect ratios structures in water.

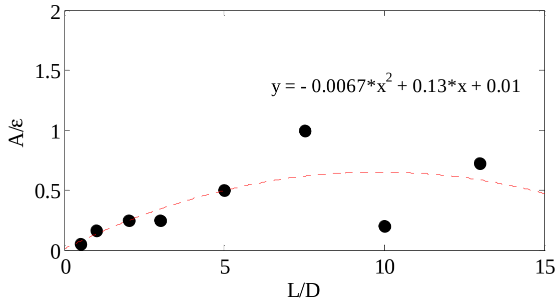

Figure 4: Model parameter ratio from modified WOM over structure aspect ratio. The dash red line represents curve fit using a single quadratic function.

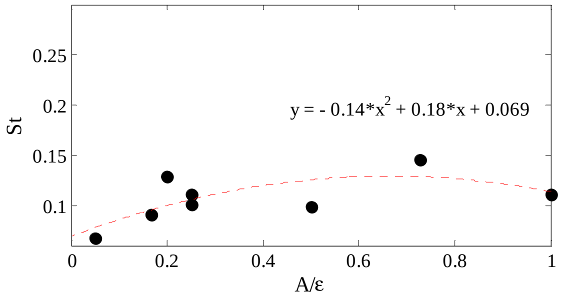

Figure 5: Strouhal Number relationship with the model parameter ratio from modified WOM. The dash red line represents curve fit using a single quadratic function.

4.0 CONCLUSION

In this paper, a phenomenological model using the wake oscillator model was developed. A wake oscillator model were examined and modified to include the influence of aspect ratio in the prediction model. Comparisons with the experimental data were provided to draw a solid conclusion on the different approaches implemented in this work. The following conclusion may be summarized.

i. The model parameters in [16] was modified accordingly to suit the experimental data. The comparison is relatively in agreement for the response amplitude of the aspect ratio investigated except for $L/D = 0.5$ where the model was a bit off at higher reduced velocities. This is due to the free surface and end effects which disturbed the vortex formation due to a reduction of the correlation length.
ii. Present analytical study has shown the important parameters affecting the results were the empirical parameters derived from the experiments together with the tuning parameters. There is no approach identified in this model to include the parameter of aspect ratio by itself. The values of the parameters used were corresponded to the influence of the aspect ratio from the experimental works.
iii. The WOM presented in this study shows a different capability to capture VIV phenomenon. Most of the model effectively captured VIV of a high aspect ratio or it was simplified as 2D problem. However, it failed to predict VIV of a cylinder with very low aspect ratio, where 3D problems involved significantly.

REFERENCE

[1]. King, R (1977). “A review of vortex shedding research and its application,” *Ocean Engineering*, Vol 4, pp 141-172.

[2]. Sarpkaya, T (2004). “A critical review of the intrinsic nature of vortex-induced vibrations,” *Journal of Fluids and Structures*, Vol 19, pp 389-447.

[3]. Bearman P, W (1984). “Vortex shedding from oscillating bluff bodies,” *Annual review of fluid mechanics*, Vol 16, pp 195-222.

[4]. Pantazopolous, M, S (1994). “Vortex-induced vibration parameters: critical review,” *13th International Conference on Offshore Mechanics and Arctic Engineering*, Vol 1, pp 199-255.

[5]. Williamson, C, H, K, Govardhan, R (2008). “A brief review of recent results in vortex-induced vibrations,” *Journal of Wind Engineering and Industrial Aerodynamics*, Vol 96, pp 713-735.

[6]. Chen, S, S (1987). “Flow induced vibration of circular cylindrical structures,” *Springer, Washington DC, US: Hemisphere Publishing Corporation.*

[7]. Blevins, R, D (1990). “Flow-induced vibration,” *New York: Van Nostrand Reinhold.*

[8]. Sumer, B, M, and Fredsoe, J (1997). “Hydrodynamics around cylindrical structures,” *Singapore: World Scientific.*

[9]. Gabbai, R, D, and Benaroya, H (2005). “An overview of modelling and experiments of vortex-induced vibration of circular cylinders,” *Journal of Sound and Vibration*, Vol 282, pp 575-616.

[10]. Bishop, R, E, D, and Hassan A, Y (1964). “The lift and drag forces on a circular cylinder in a flowing fluid,” *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol 277 (1368), pp 32-50.

[11]. Hartlen R, T, and Currie, G (1970). “Lift-oscillator model of vortex-induced vibration,” *Journal of the Engineering Mechanics Division*, Vol 5, pp 577-591.

[12]. Skop, R, A, and Griffin, O, M (1973). “A model for the vortex-excited resonant response of bluff cylinders,” *Journal of Sound and Vibration*, Vol 27(2), pp 225-233.

[13]. Iwan, W, D, and Blevins, R, D (1974). “A model for vortex induced oscillation of structures,” *Journal of Applied Mechanic*, Vol 41, pp 581-586.

[14]. Landl, R (1974). “A mathematical model for vortex-excited vibrating bluff-bodies,” *Journal of Sound and Vibration*, Vol 42, pp 219-234.

[15]. Griffin, O, M (1980). “Vortex-excited cross-flow vibrations of a single cylindrical tube,” *ASME Journal of Pressure Vessel Technology*, Vol 102, pp 158 – 166.

[16]. Facchinetti, M, L, de Langrea E, and Biolleyb, F (2004). “Coupling of structure and wake oscillators in vortex-induced vibrations,” *Journal of Fluids and Structures*, Vol 19, pp 123–140.

[17]. Simiu, E, and Scanlan, R, H (1986). “Wind effects on structures,” *Wiley-Interscience, New York*, Second Edition.

[18]. Goswami, I, Scanlan, R, H, Jones, N, P (1993). “Vortex-induced vibration of circular cylinders-part 2: new model,”*ASCE Journal of Engineering Mechanics*, Vol 119, pp 2288–2302.

[19]. Sarpkaya, T (1978). “Fluid forces on oscillating cylinders,” *Journal of Waterway Port Coastal and Ocean Division ASCE*, WW4, Vol 104, pp 275-290.

[20]. Griffin, O, M, Koopman, G, H (1997). “The vortex-excited lift and reaction forces on resonantly vibrating cylinders,” *Journal of Sound and Vibration*, Vol 54, pp 435–448.

[21]. Birkhoff, G, and Zarantanello, E, H (1957). “Jets, wakes and cavities,” *Academic Press*, New York.

[22]. Nayfeh A, H (1993). “Introduction to perturbation techniques,” *Wiley*, New York.

[23]. Rahman, M. A. A., and Thiagarajan, K. (2013). “Vortex-Induced Vibration of Cylindrical Structure with Different Aspect Ratio,” Proceedings of the 23rd International Offshore and Polar Engineers, Alaska, USA.

[24]. Gonçalves, R, T, Franzini, G, R, Rosetti, G, F, Meneghini, J, R, and Fujarra, A, L, C. “Flow around circular cylinders with very low aspect ratio,” *Journal of Fluids and Structures*, http://dx.doi.org/10.1016/j.jfluidstructs.2014.11.003.