

# Helmholtz Equation Applied to the Vertical Fixed Cylinder in Wave Using Boundary Element Method

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## ABSTRACT

Helmholtz equation is employed to the wide variety of engineering problems. This paper presents using Helmholtz equation to the vertical fixed cylinder against regular wave by the boundary element method (BEM). The results are included the pressure and forces on the cylinder. The present results are compared with available data, and show in good agreement. The results show that up to a certain dimensionless wave length, the wave force has a direct relation with the dimensionless wavelength, while after the force decreases with the increase of the dimensionless wave length.

**KEY WORDS:** *Wave-cylinder interaction, Boundary element method, Helmholtz equation.*

## NOMENCLATURE

$k$	Wave Number
$\delta(r)$	Delta Function
$G$	Green's Function
$r$	Distance Between Source Point (p) and field point (q)
$H_0^{(1)}$	Hankel Function of First Kind of Zero Order
$J_0$	Bessel Function of the First Kind
$Y_0$	Bessel Function of the Second Kind
$\phi^{Diff}$	Velocity Potential of Diffracted Wave

$\phi^{In}$	Velocity Potential of Incident Wave
$\phi^{Total}$	Total Velocity Potential
$g$	Gravity Acceleration
$H_w$	Wave Height
$\omega$	Wave Frequency
$\alpha$	Angle of the Incident Wave
$\Gamma_{BS}$	Body Surface Boundary
$\Gamma_{FS}$	Free Surface Boundary
$\Gamma_{SB}$	Sea Bottom Boundary
$\Gamma_{Inf}$	Infinity Boundary
$\beta$	Interior Angle of the Corner
$P$	Wave Pressure
$F$	Wave Force
$h$	Water Depth
$R$	Radius of the Cylinder

## 1.0 INTRODUCTION

The Helmholtz equation or reduced wave equation is widely employed in different fields of science. One of the most important applications of the Helmholtz equation is in wave-structure interaction problems. The aim of this paper is to determine wave forces on a vertical cylinder using the BEM. Since the main focus of this research is on the wave-cylinder interaction problem, in the remaining part of this section, first the available data on the application of the Helmholtz equation in different problems is reviewed and then a special paragraph is dedicated to review the application of the Helmholtz equation in wave-structure interaction.

There are several researches about the application of the Helmholtz equation for the solution of the problem in a broad area of sciences. Marin used BEM with a constrained least-squares minimization, to solve the inverse geometric problem of detection of a single and two circular cavities in Helmholtz-type

equations from boundary data [1]. Chen et al. used BEM for numerical study of the occurrence of irregular frequency (fictitious frequency) for the exterior Helmholtz equations with mixed-type boundary conditions [2]. Tamioka and Nishiyama present a gradient field representation by an analytical regularization of hyper singular boundary integral for a two dimensional Helmholtz equation [3]. Bin-Mohsin and Lencic developed the method of fundamental solutions (MFS) for solving Helmholtz- type elliptic partial differential equations in composite materials [4]. Dogan et al. developed a mesh-free method for the solution of Helmholtz equation by using the radial basis integral equation method (RBIEM) for the analysis of the acoustic pressure in a sonoreactor [5]. Darbas et al. used an iterative solution to three dimensional sound- hard acoustic scattering problems at high frequency considering the combined Field Integral Equation (CFIE) [6]. Casenave et al. derived coupled BEM- FEM formulations for the convected Helmholtz equation modeling linear acoustic propagation at a fixed frequency in a subsonic flow around a scattering object [7]. With the main focus on the omitting singularity from BEM's domain integrals, Hosseinzadeh and Dehghan developed a new idea to solve linear poisson's and Helmholtz equations with variable coefficient by the use of BEM [8]. Loeffler et al. applied a direct interpolation technique using radial basis function to the BEM integral term, inertia in the Helmholtz equation to determine the free vibration frequencies and amplitudes from an eigenvalue problem solution [9]. Ghassemi et al. used the dual reciprocity boundary element method (DRBEM) for the acoustic propagation inside the silencer in the three dimensional form of the Helmholtz Equation [10].

BEM solution of the Helmholtz equation of the problem of wave forces on vertical cylinders was first presented by Au and Brebbia [11]. Zhu studied the combined diffraction and refraction of water waves propagating around offshore structures over a seabed with a variable depth by the DRBEM method [12]. Kim et al. used BEM for the analysis of wave force and run up on vertical circular cylinders [13]. Kim and Cao used BEM to calculate the wave force on plural vertical cylinders [14]. Ketabdari et al. applied three dimensional BEM for modeling of the interaction between waves and a number of tandem fixed cylinders [15]. Chuang et al. studied the wave scattering by a concentric cylindrical system by a dual boundary element model coupled with the dual reciprocity method [16].

The rest of the paper is organized in the following sections. The Helmholtz equation and an example of the application of this equation in wave-structure interaction are described in sections 2 and 3, respectively. The results of BEM are presented in section 4. Finally, conclusions are given in section 5.

## 2.0 HELMHOLTZ EQUATION

In a two dimensional domain  $\Omega$  with the boundary  $\Gamma$ , the Helmholtz equation takes the following form

$$\nabla^2 \phi + k^2 \phi = 0 \quad (1)$$

With this equation and the related boundary conditions, a BEM problem is constituted.

The Green's function should satisfy the Eq. (2),

$$\nabla^2 G + k^2 G = \delta(r) \quad (2)$$

The Green equation is obtained as follows

$$G = \frac{1}{4i} H_0^{(i)}(kr) = \frac{1}{4i} [Y_0(kr) + iJ_0(k)] \quad (3)$$

In this research,  $Y_0$  and  $J_0$  series are approximated by adding the five first sentences together.

## 3.0 WAVE-STRUCTURE INTERACTION

The problem domain is shown in Figure 1. Total velocity potential can be described by the sum of incident and diffracted waves

$$\phi^{Total} = \phi^{Diff} + \phi^{In} \quad (4)$$

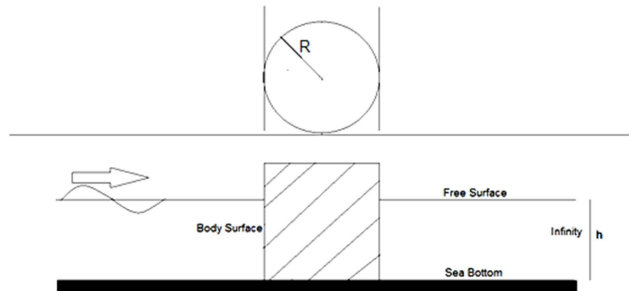


Figure 1: The Problem Domain.

The potential of the linear incident wave could be described by Eq.5

$$\phi^{In} = -\frac{igH_w}{2\omega \cosh kh} \cosh kZ \exp(i(K_1x + K_2y - \omega t)) \quad (5)$$

Where  $Z = z + h$  and  $K_1$  and  $K_2$  are defined by Eq.6

$$K_1 = k \cos \alpha \quad K_2 = k \sin \alpha \quad (6)$$

If the cylinder cross section is not changed in vertical direction, the incident wave potential can be simplified according to the following equation

$$\phi^{In} = \varphi^{In} \cosh kZ \quad (7)$$

$$\varphi^{In} = -\frac{igH_w}{2\omega \cosh kh} \exp(i(K_1x + K_2y - \omega t)) \quad (8)$$

The governing equation on the diffracted wave could be expressed by the Eq. (9),

$$\nabla^2 \phi^{diff} = 0 \text{ in } \Omega(x, y, z) \quad (9)$$

The boundary conditions are

$$\frac{\partial \phi^{Diff}}{\partial n} + \frac{\partial \phi^{In}}{\partial n} = 0 \quad \text{on } \Gamma_{BS} \quad (10)$$

$$\frac{\partial \phi^{Diff}}{\partial n} = 0 \quad \text{on } \Gamma_{SB} \quad (11)$$

$$\frac{\partial \phi^{Diff}}{\partial n} - ik\phi^{Diff} = 0 \quad \text{on } \Gamma_{inf} \quad (12)$$

$$\frac{\partial \phi^{Diff}}{\partial n} - \frac{\omega^2}{g}\phi^{Diff} = 0 \quad \text{on } \Gamma_{FS} \quad (13)$$

It can be shown that the Green's equation (Eq.3) automatically satisfies the boundary conditions (11-13) [11]. Only the body surface boundary condition (Eq.10) should be considered in the BEM solution.

According to the Green's Theory, the boundary integral equation is as Eq. (14),

$$\varepsilon_i \phi_i^{Diff} + \sum_{j=1}^N \int_{\Gamma_j} \frac{\partial G}{\partial n} d\Gamma \{ \phi_j^{Diff} \} = - \sum_{j=1}^N \int_{\Gamma_j} G d\Gamma \left\{ \frac{\partial \phi_j^{In}}{\partial n} \right\} \quad (14)$$

Where

$$\varepsilon_i = \begin{cases} 1 & i \in \Omega \\ 0.5 & i \in \Gamma \text{ (smooth)} \\ \beta/2\pi & i \in \Gamma \text{ (not smooth)} \\ 0 & i \notin \Omega \end{cases} \quad (15)$$

where  $\beta$  is the interior angle of the corner at  $i$ . Consequently,  $\phi^{Diff}$  is found by solving the following linear system

$$[H]\{\phi^{Diff}\} = [L]\left\{\frac{\partial \phi^{In}}{\partial n}\right\} \quad (16)$$

Using linearized Bernoulli's equation, the wave pressure and wave force on the structure can be obtained by

$$P = -\rho \frac{\partial \phi^{Total}}{\partial t} \quad (17)$$

$$F = i\rho\omega \frac{\tanh kh}{k} \int_{\Gamma_{Bs}} [n]\phi^{Total} d\Gamma \quad (18)$$

#### 4.0 PROBLEM SETUP

The cylinder radius is  $R = 10$ , the wave height is  $H_w = 1.0$  and the water depth is  $h = 5R$ . The wave number,  $k$ , is calculated by non-dimensional number  $kR$  for different values of  $kR$ . According to the linear wave theory, the wave frequency  $\omega$  is calculated by the following equation

$$\omega = \sqrt{gk \tanh kh} \quad (19)$$

Solution of the problem is done by using twenty constant type elements which are distributed on the surface of the cylinder. Using Eq. (14), the problem is discretized to linear system of equations and the value of the diffraction potential on each element is calculated. Then the Eqs (17) and (18) are used for the calculation of the pressure distribution on the surface of the cylinder and the wave force, respectively.

#### 5.0 RESULTS

In this section, the BEM results of the wave pressure and wave force are presented. The wave pressure is composed of Real and Imaginary parts. As it is shown in Figures 2 and 3, the maximum and the minimum mode of the wave pressure are derived from the Real and Imaginary parts of total wave pressure.

For plotting the wave force, usually the dimensionless forms of the force are plotted. In this research, we use two non-dimensional forms of wave force defined by Eqs. (19) and (20)

$$F_1 = \frac{F}{\frac{1}{2}\rho g H_w R h \frac{\tanh kh}{kh}} \quad (19)$$

$$F_2 = \frac{F}{\frac{1}{2}\rho g H_w R^2} \quad (20)$$

Figure 4 shows that there is a steep increase in  $F_1$  with the dimensionless wave length up to  $kR < 0.8$ . After that,  $F_1$  decreases with a slight slope.

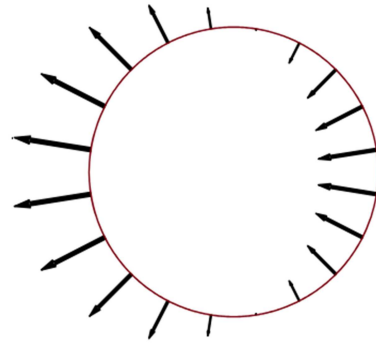


Figure 2: Pressure distribution on cylinder at maximum mode at  $\alpha = 0$ .

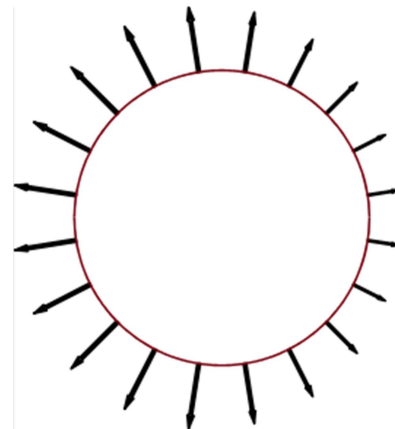


Figure 3: Pressure distribution on cylinder at minimum mode at  $\alpha = 0$ .

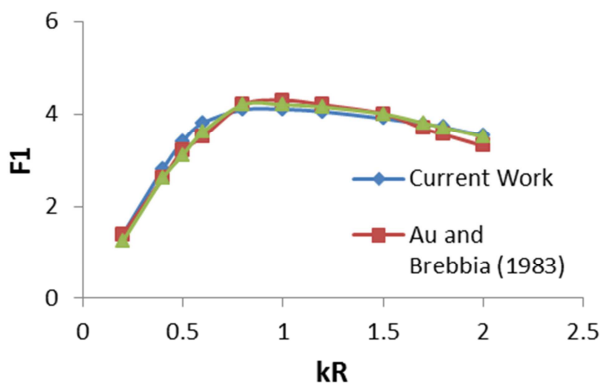


Figure 4: Force  $F_1$  against  $kR$ .

As it is shown in the Figure 5, when  $kR$  (the dimensionless wave length) is equals to 0.5,  $F_2$  has its maximum amount. After that, any increase in  $kR$  would be resulted in a decrease in  $F_2$ .

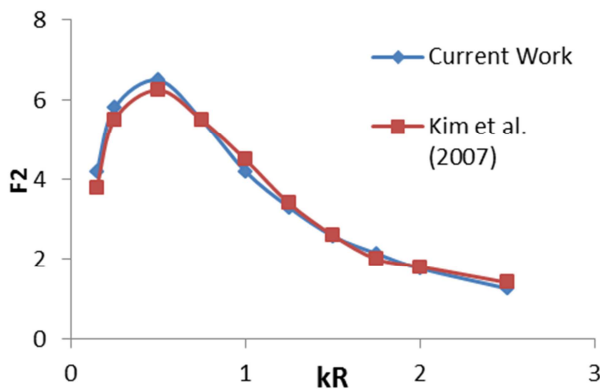


Figure 5: Force  $F_2$  against  $kR$ .

## 6.0 CONCLUSIONS

In this paper, Helmholtz equation employed to the vertical cylinder in waves using BEM. Two different definitions for horizontal wave force were introduced and the effect of the dimensionless wave length on these forces was studied. It is shown that by increasing  $kR$ , the wave force is first increased up to a certain amount of  $kR$  and then decreased. The results were compared to results of available validated papers and a good agreement was obtained. These outcomes prove that the BEM could be used for accurate estimation of Helmholtz type problems in different kind of engineering fields.

For future studies, it is proposed to study the following topics

- The effect of different wave angles
- The effect of different  $h/R$  ratios
- The effect of using higher order elements instead of constant elements
- The effect of using other wave theories such as Stokes wave theories.

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