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Finite Element-Based Optimization of Cross-Section Area on 2D Truss Structure with 10-members and 12-members Using Nonlinear Programming Method

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ABSTRACT

Designing a structure requires good planning to obtain optimal results. Some things that need to be considered in designing structures are the materials used, the dimensions of the structure, and others. By optimizing the dimensions of the structure, the use of materials can be minimized so that production costs can be reduced. The experiment requires additional cost and time. Therefore, finite element software can be used to design the structure optimally in compliance with the constraints set according to the requirements. Previous studies have carried out cross-section area optimization of structures with maximum allowable stress limits. In this research, the optimization of 2D truss cross-section area with maximum volume constraint is developed using MATLAB software with fmincon function. The structure is modelled in 2 shapes: a 2D truss with ten members and a 2D truss with twelve members. The optimum area is obtained from the simulation results, which are almost the same in both structures, while the lower stress value is obtained in the ten-member 2D truss structure. The maximum stress (tensile stress) on the 12-member 2D truss occurs on element 12 at 3484.28 psi, and then the minimum stress (compressive stress) occurs on elements 3, 4, 8, and 9 at -3472.10 psi. The maximum stress (tensile stress) on the 10-member 2D truss structure occurs in elements 1 and 9 at 3472.10 psi, and the minimum stress (compressive stress) occurs in element four at -3479.77 psi.

KEYWORDS: *Truss 2D, Finite element, Constraint, Optimization, Fmincon.*

NOMENCLATURE

v^U	Upper limit on volume
F	Force
L	Element length
U	Displacement
A	Cross section area
σ	Stress
A^U	Upper limit on cross-section area
A^L	Lower limit on cross-section area
K	Stiffness
∇	Gradient

1.0 INTRODUCTION

When designing a structure, structure design planning is carried out according to the loading conditions on the structure. It is essential to consider the dimensions of the structure, structure material, structure shape, and others by the requirements needed. An example of using structures in the mechanical field is in vehicle chassis [1][2]. Several numerical programs have also been developed for structural analysis and optimation of the internal environment [3],[4],[5].

Cost can be reduced by optimizing the cross-sectional area to reduce the required material while maintaining requirements such as allowable stress, maximum deflection, and others [6],[7]. Structure optimization can be carried out by experiments and simulations using finite element programs [8],[9]. However, experiments require more costs and time, while simulations using finite element programs can reduce costs and processing time.

Many researchers have studied the optimization of structures using finite element programs. Among them is the "Multi-Objective Particle Swarm Optimization" (MOPSO) method to optimize the size and shape of the structure [10], where the structural constraints specified are the maximum and minimum area and maximum stress so that the optimal area and shape of the structure are obtained. Then, the "Virtual Work

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Optimization Method (VWOM)" using the MATLAB finite element program, where this researcher models 2D trusses structures with the specified constraints as the maximum and minimum area, maximum stress, and maximum deflection so that the optimal area is obtained [11].

In this study, cross-sectional area optimization was carried out to minimize the displacement of the structure using the MATLAB finite element program with the "*fmincon*" method. The structure is modelled as a 2D truss with 10-member and 12-member. The constraints specified are maximum volume, maximum area, and minimum area. Then, the area optimization results of the 10-member 2D truss are compared with the 12-member 2D truss.

2.0 THEORETICAL BACKGROUND

Finite element is a common method used for structure analysis. Finite element discretization of these differential equations takes the form of linear simultaneous equations for U (displacement):

$$K(x)U = F(x) \qquad (1)$$

where K is a ($ndof \times ndof$) square stiffness matrix and F is a($ndof \times 1$) load vector.

2.1 Define Variables with Direct Method

Efficient computation of gradients is important owing to the size of the matrices. Specifically, consider an implicit function q, which can represent a cost or constraint function as:

$$q = q(x,U) \qquad (2)$$

Let x^0 be the current design. To evaluate the gradient at x^0 analytically, it can be obtained by the direct method, i.e. by differentiating equation (2), thus obtained:

$$\nabla q \equiv \frac{dq}{dx} = \left[\frac{dq}{dx_1}, \frac{dq}{dx_2}, \ldots, \frac{dq}{dx_n}\right] \textit{evaluated at } x^0 ,$$

$$\frac{dq}{dx_i} = \frac{\partial q}{\partial x_i}, \frac{dq}{dU}\frac{dU}{dx_i}, \quad i = 1, \ldots, n \qquad (3)$$

Partial differentiation concerning x_i is done while keeping U fixed and vice versa. In equation (3), the derivatives $\partial q/\partial x_i$ dan $\partial q/\partial U$ are readily available from the definition of q. For example, if:

$$q = 3x_1 U_3 + 2x_2^3 + 2.5\,U_5^2,\ n = 3,\ ndof = 6 \text{ , then:}$$

$$\frac{\partial q}{\partial x} = \left[3\,U_3^0, 6\left(x_2^0\right)^2, 0\right], \text{ and } \frac{\partial q}{\partial U} = \left[0, 0, 3x_1^0, 0, 5\,U_5^0, 0\right]$$

where U^0 is the displacement vector at the current point, obtained by solving $K(x^0)U^0=F(x^0)$. The main hurdle in equation (3) is to evaluate the displacement derivatives dU/dx_i This is obtained by differentiating equation (1) for x_i at x^0:

$$K\left(x^0\right)\frac{dU}{dx_i} + \frac{\partial K}{\partial x_i}U^0 = \frac{dF}{dx_i}$$

$$K\left(x^0\right)\frac{dU}{dx_i} = -\frac{\partial K}{\partial x_i}U^0 + \frac{dF}{dx_i}, \quad i = 1, \ldots, n \qquad (4)$$

The direct method consists of solving for dU/dx_i from equation (4) and then substituting this into equation (3). Regarding the computational effort needed to obtain it, note that a finite element analysis has to be performed; hence, equation (1) has already been solved. It means that the factorized K matrix is available. We can use this same factorized matrix in equation (4). Thus, the solution of equation (4) is analogous to solving a matrix system with n right-hand side “load” vectors. Thus, the operations count may be summarized as involving.

Direct method: n right-hand side vectors (5)

Each right-hand side vector involves $ndof^2$ operations (an operation being an addition together with a multiplication – the multiplication of a $1 \times ndof$ vector with a $ndof \times 1$ vector will thus involve $ndof$ operations) [12]. Thus, the direct method involves $nndof^2$ operations.

The stress in the structure can be determined from equation (6):

$$\sigma = \frac{f_{2x}}{A}, \quad where\text{: } f_{2x} = \frac{AE}{L}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}u_{1x}\\ u_{2x}\end{Bmatrix} \text{, then obtained:}$$

$$\{\sigma\} = \frac{E}{L}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}u_{1x}\\ u_{2x}\end{Bmatrix}, \ or\text{: } \{\sigma\} = \frac{E}{L}[T]\{D\}$$

Which can be simplified in the form:

$$\{\sigma\} = [C']\{D\} \qquad (7)$$

where:

$$\left[C'\right] = \frac{E}{L}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}c & s & 0 & 0\\ 0 & 0 & c & s\end{bmatrix}$$

f_{2x} is an axial force that works at the end nodal of an element, c is the result of dx/L, and s *is* the result of dy/L [13].

2.2 Nonlinear Programming Methods (NLP)

The general problem of structures has constraints on displacement, stress, volume, and resonant frequency. It can be solved by directly interfacing nonlinear programming (NLP) algorithms with finite element (FE) codes. NLP codes in programming include ZOUTEN, GRG, SQP, PENALTY, Matlab "*fmincon*", Excel SOLVER and so on. Interfacing these two codes is easy; the same names for variables should be avoided. For example, the NLP code can use $\boldsymbol{x}$ for variables, and the FE code can use $\boldsymbol{x}$ for nodal coordinates. However, it is necessary to be concerned about the computational process because each function call is an FE analysis[14]. In this research, the Matlab NLP code *fmincon* will be used to determine the optimum area of the structure. The *fmincon* is a function in MATLAB that solves nonlinear programming cases by finding the optimum value of the problem. The *fmincon* is a

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constraint-based optimization method. The constraints (upper limit/maximum and lower limit/minimum) variables are determined.

3.0 METHODS

3.1 Structure Parameters

The material used in this study is aluminium alloy 6061-T6 (SS) with the parameters and constraints of the 2D truss structure 10-member and 12-member shown in Table 1.

Table 1 2D Truss Structure Parameters

Parameter	10-member 2D truss	12-member 2D truss
External force (*F*)	10^5 lb	10^5 lb
modulus of elasticity (*E*)	10^7 psi	10^7 psi
Yield strength (σ_u)	39885 psi	39885 psi
The upper limit on volume (V^U)	82947 in^3	82947 in^3
The lower limit on the area (A^L)	10^{-6} in^2	10^{-6} in^2
The upper limit on the area (A^U)	100 in^2	100 in^2

Figure 1 shows the modeling of the 10-member 2D truss structure and Figure 2 shows the modeling of the 12-member 2D truss structure.

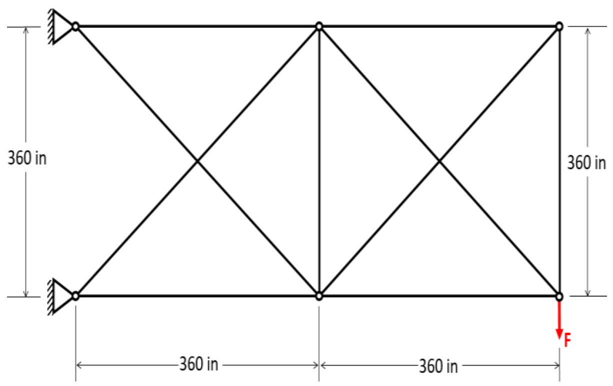

Figure 1: 10-member 2D truss structure

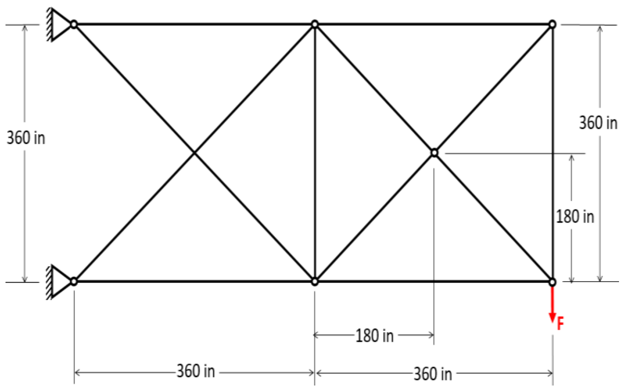

Figure 2: 12-member 2D truss structure

3.2 Finite Element (FE) Modelling of Structures

Figure 3 shows the modelling of a 10-member 2D truss structure. The finite element model of this structure is divided into 10 elements with 6 nodes so that the total degrees of freedom of the structure become 12. The structure is clamped at nodes 1 and 4 so that the displacement at nodes 1 and 4 is zero.

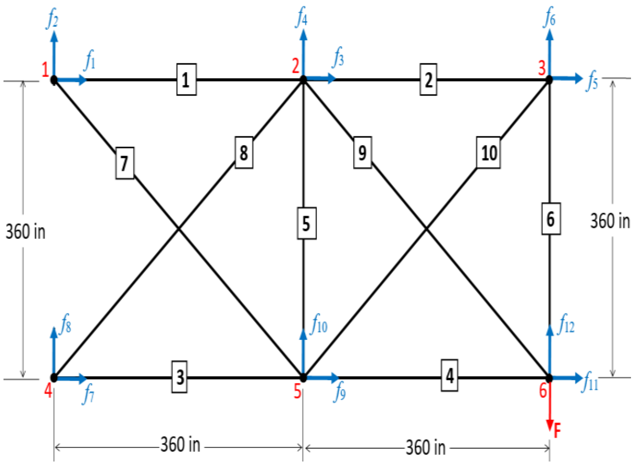

Figure 3: Finite element modelling of 10-member 2D truss structure

Furthermore, Figure 4 shows the modelling of the 12-member 2D truss structure. The finite element model of this structure is divided into 12 elements with 7 nodes so that the total degrees of freedom of the structure become 14. Just like the 10-member 2D truss structure, in the 12-member 2D truss structure, the structure is also clamped at nodes 1 and 4 so that the displacement at nodes 1 and 4 is zero.

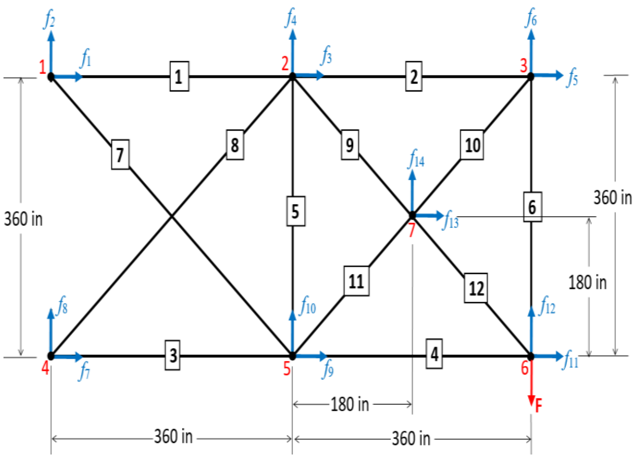

Figure 4: Finite element modelling of 12-member 2D truss structure

From Figures 3 and 4, it can input the 2D truss data of 10-member and 12-member in different MATLAB editors. Then, compile the local and global stiffness matrices to determine the optimum area of the 2D truss using the MATLAB function "*fmincon*". The last step, determine displacement and the stress on each element of the 10-member and 12-member 2D truss.

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4.0 RESULT AND DISCUSSION

4.1 Area Optimization Results Using MATLAB *fmincon*

The optimum areas of the running program results are shown in Figure 5 and Figure 6.

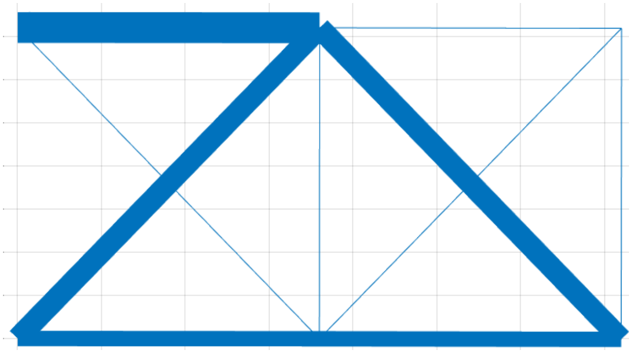

Figure 5: Area Optimization Results of 10-member 2D Truss using MATLAB *fmincon*

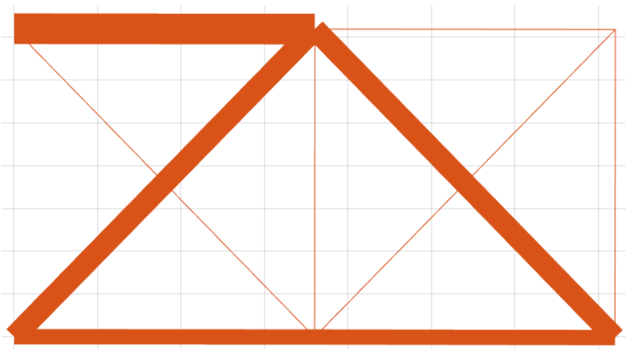

Figure 6: Area Optimization Results of 12-member 2D Truss using MATLAB *fmincon*

To obtain the optimum results, 1828 FE calls were required on the 10-member 2D truss, and 3006 FE calls on the 12-member 2D truss. The optimum area value of each element of the 10-member and 12-member 2D truss structure in Figure 5 and Figure 6 can be viewed in Table 2.

Table 2: Optimum area value of the 2D truss structure

Element/ member	Optimum area (A) (in^2)	
	10-member truss	12-member truss
1	57.602	57.642
2	10^{-6}	0.0012
3	28.801	28.786
4	28.801	28.737
5	10^{-6}	0.00028
6	10^{-6}	0.00074
7	10^{-6}	0.0015
8	40.731	40.773
9	40.731	40.835
10	10^{-6}	0.0022
11	-	0.0013
12	-	40.587

The maximum volume of a 10-member 2D truss structure was 82947 in^3. This result is equal to the maximum volume of the 12-member 2D truss structure, which was 82947 in^3.

4.2 Optimum Displacement Result of the Structure

The optimal (minimum) displacement of each node in the structure from the results of the 2D truss structure area optimization using MATLAB *fmincon* is shown in Table 3.

Table 3: Optimal displacement (U) of the structure

nodal	10-member 2D truss		12-member 2D truss	
	x (in)	y (in)	x (in)	y (in)
1	0	0	0	0
2	0.125	-0.375	0.125	-0.375
3	0.211	-0.914	0.211	-0.857
4	0	0	0	0
5	-0.125	-0.357	-0.125	-0.341
6	-0.25	-1	-0.25	-1
7	-	-	0.022	-0.602

Table 3 shows that the maximum displacement occurs in the y-axis of nodal six by 1 in the displacement moves downward (negative y-axis direction) to be negative according to the load applied to nodal 6 (F) of 105 lb. This result was equal in both 10-member and 12-member 2D trusses. In both structures, considerable displacement occurs in the y-axis due to the load applied to node 6 in the downward direction (negative y-axis). Adding a pin (node 7) in the 12-member 2D truss structure causes displacement in the y-axis of nodes 3 and 5 to decrease because displacement in the structure is distributed to the added node (node 7).

4.3 Optimum Stress Result of the Structure

In the last step, the stress calculation is obtained by adding/inputting the stress value equation (equation (6)) into the finite element program. The results are shown in Table 4. Table 4 shows that the maximum stress (tensile stress) on the 12-member 2D truss occurs on element 12 at 3484.28 psi, and then the minimum stress (compressive stress) occurs on elements 3, 4, 8, and 9 at -3472.10 psi. The maximum stress (tensile stress) on the 10-member 2D truss structure occurs in elements 1 and 9 at 3472,10 psi and the minimum stress (compressive stress) occurs in element 4 at -3479.77 psi.

Table 4: Stress optimum value of the 2D truss structure

Element/ member	stress (σ) (psi)	
	10-members	12-members
1	3472.10	3469.58
2	2389.95	2403.50
3	-3472.10	-3474.01
4	-3472.10	-3479.77
5	-496.50	-947.11
6	2387.83	3960.44
7	3223.85	2992.63
8	-3472.10	-3468.41
9	3472.10	3463.16
10	-3067.51	-1843.99
11	-	-3166.40
12	-	3484.28

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The stress in these two structures is still within safe limits because the maximum stress value (tensile stress) is well below the yield strength value of the material, where the yield strength value of aluminium alloy 6061-T6 (SS) is 39885 psi. The average structure stresses are shown in Table 5.

Table: 5 Average stress of 2D truss structure

Stress	10-members (psi)	12-members (psi)
Tensile stress	2989.16	3295.60
Compressive stress	-2796.06	-2729.95

The most important factor for designing stresses in static structures was the yield strength limit of the material, which needs to be considered was tensile stress. Table 5 shows that the average stress value of the 10-member 2D truss structure was smaller than the average stress of the 12-member 2D truss structure. Likewise, with the maximum stress on each element, the maximum stress value (tensile stress) on each element of the 10-member 2D truss structure was smaller than the maximum stress value on each element of the 12-member 2D truss structure.

5.0 CONCLUSION

The simulation results of area optimization using MATLAB *fmincon* conclude that the optimum area in the 2D truss structure of 10-member and 12-member is almost close (similar tendency). The comparison is shown in Figures 5 and 6. It means that adding pins to the structure (Figure 2) does not significantly affect changing the optimum area value of the structure. However, FE calls on a 12-member 2D truss are almost twice as much as FE calls on a 10-member 2D truss, so the cost for a 12-member 2D truss will be higher. Adding pins to the structure (Figure 2) causes the displacement at other nodes to decrease slightly due to distribution to node 7 (the added pin). Stress in the structure increases due to the addition of pins at node 7 (Figure 2). Therefore, it can be concluded that the design of the 10-member 2D truss structure (Figure 1) more effectively reduces the stress in the structure than the 12-member 2D truss structure (Figure 2).

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