

Buckling and Deflection of Ice Sheet Caused by Ship

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ABSTRACT

The movement of ice ship gradually gives load to the ice sheet, along with that, the ice sheet will react in proportion to the load of the ship. The phenomenon of interaction between ice and hull of ship has been studied by researchers through empirical mathematical simulation and experiment. This paper discusses the phenomena of ice sheet buckling and deflection caused by a ship sailing in ice level. The ice sheet buckling and deflection is analyzed based on using Finite Element Method based on Euler and Cantilever methods. The methods were described from general deflection equation for a beam. Simulation was carried out using Abaqus running ahead and astern at several speeds of 0.4m/s, 0.5m/s and 0.6m/s.

KEY WORDS: *Buckling Ice; Deflection of Ice; Ice Bow; Bulbous Bow; Ice Ship; Finite Element Method*

NOMENCLATURE

FEM	Finite Element Method
CEL	Couple Eulerian Lagrangian
DAT	Double Acting Tanker

1.0 INTRODUCTION

Resistance of ships at the ice level is a very basic and important field in the early stages in ice class ship design because it is closely related to ship propulsion and determines power of ship engine. Determining the ship resistance in the level ice is more complex than in the open water due to the changing characteristic properties of ice and icebreaking phenomena. Ice resistance is defined as the time average of all longitudinal forces due ship-ice interactions.

The phenomenon of interaction between ice and ship has been studied by researchers through empirical mathematical simulation. The empirical mathematical can be used to determine the power needed by a ship to travel through the ice sheet on certain characteristics according to the desired speed. They can also be used to gain insight into the influence of the hull form on ice resistance. Lewis et.al (1970) proposed semi-empirical which was developed based on a number of experimental data of ice breakers which included full scale testing on lakes and sea ice and test the model in fresh ice and sea [1]. The method has a semi-empirical relationship between ice resistance and the parameters that characterize ships and ice sheets. The empirical formula consists of ice breaking, friction, ice buoyancy and momentum. Crago et al. (1971) described a set of model test in "wax-type" ice on 11 icebreakers [2]. Enkvist (1972) studied three icebreakers: Moskva-class, Finn carrier, and Jelppari [3]. Milano (1973) made a significant advance in the purely theoretical prediction of ship performance on ice based on conservation energy [4]. Vance (1975) obtained an "optimum regression equation" from five sets of model and full-scale data, of the Mackinaw same data as used by Lewis et.al (1970) [5, 6]. Lindqvist (1989) developed a formula to calculate ice resistance based on many full scale tests in the Bay of Bothnia [7]. Keinonen et al. (1996) did research on resistance of icebreaking vessels in level ice and developed a formula based on results of a study of escort operations involving five icebreaking vessels [8]. Daley, et.al (1997 & 1998) proposed a level ice resistance formula with some empirical parameters by developing Lindqvist's formula [9, 10]. Jaswar (2002 & 2005)

proposed a method to predict ice resistance of a ship running in unfrozen and frozen ice channels and level ice [11, 12]. Su et al. (2010) stated that is often difficult to make the good relation between model scale test to full scale condition [13]. This is the current weakness in the design of an ice class ship. Jeong et al. (2010) proposed new ice resistance prediction formula for standard icebreaker model using component method of ice resistance and also predicted the model test results to full-scale using calculated non-dimensional coefficients [14]. Continuing the previous research, Tan et al. (2013 & 2014) studied the effect of the propeller-hull-ice interaction of a dual-direction ship during running astern obtained from model tests on applied to the numerical procedure [15, 16]. The model tests were conducted by Leiviska (2004) on a model of the M/T Uikku to investigate the propeller-hull-ice interaction [17]. The numerical procedure is in turn used as a performance prediction tool to supplement the model test data to investigate the thrust deduction in ice. Hu.et.al (2015 & 2016) discussed several numerical methods based on Lindqvist, Keinonen, Riska and Jeong to calculate ice resistance and then calculated results are compared against model test results [18, 19]. The prediction of ice resistance of icebreakers has different accuracy and also the empirical methods were under estimates for double acting tanker. Jeong.et.al (2017) presented a semi-empirical model to predict ship resistance in level ice based on Lindqvist's model [20]. Contact between the ship and the ice was assumed a case of symmetrical collision. Efi et.al (2014, 2016, 2017 & 2018) has studied performance double acting ship during running in level ice [21-27]

Design of an ice class ship requires considering the performance, adequate hull and strength of machinery and good functioning of the ship in ice condition and open water condition. The ice bow economically has inescapable disadvantage during sailing in open water due to higher resistance compared with a common bow. Researchers have proposed a Double-Acting Tanker which can sail astern functionally as an icebreaker in ice bound and ahead in open water. The stern part of DAT is specifically designed to be strong enough to break ice and pod propulsion systems. It is generally recognized phenomena of hull-ice-propeller is very complex and difficult to be understand, therefore model and full scale ice tests has been conducted to determine ice resistance of Double Acting Tanker. This paper discusses on effect of bulbous bow on ice resistance of conventional bow ship sailing in ice bounded condition which is analysed using Finite Element Method.

2.0 FUNDAMENTAL OF ICE SHEET BUCKLING

2.1 Bulbous Bow

Concept of double acting ship has started developed since 1990 by Kvaerner Masa-Yards Arctic Technology Centre which known as Aker Arctic Technology Inc., a Finnish company. The idea to build ice breaking merchant ship appeared to eliminate ice breaker as assistance when merchant ship sailing in ice conditions as mentioned by Kubiak (2014) [27]. Double acting ship was designed to run ahead in open water and astern in ice conditions. Design of ice-going ships requires considering the performance, adequate hull and strength of machinery and good functioning of the ship in ice condition and open water condition. The structure

of double acting ship has been improved by increasing the strength of structure to ensure the hull structure can withstand with ice resistance while break the ice.

The stem hull design of double acting ship differs from common ships. The common ships have a bulbous bow at the head of ship as shown in Figure 1. The main function of bulbous bow is to reduce the drag force that it was an effect of wave making resistance while ship moving ahead in open water. Therefore, the resistance of ship will reduce that can make increasing speed and improve stability of a ship.

The combined influence of a subsurface bulb and a conventional bow on wave formation where the wave created by the bulb cancels that created by the conventional bow is shown Figure 1. Description of the figure is as follows: profile of bow with bulb is indicated by no.1, profile of bow without bulb is indicated by no.2, wave created by bulb is indicated by no.3, waves created by conventional bow is indicated by no.4, and waterline and region of cancelled waves is indicated by no.5.

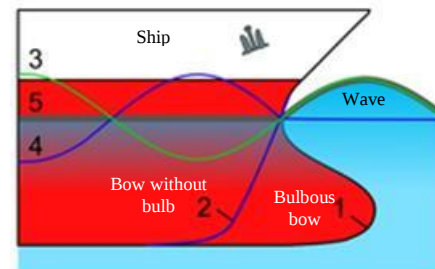


Figure 1: Bulbous bow for common tanker

By referring to Figure 1, the bulbous bow has several important advantages as follows:

1. The bulbous bow reduces the bow wave, due to the wave generated by the bulb itself
2. The ship more efficient in terms of resistance, reducing the installed power requirements and so the fuel oil consumption.
3. Works as a robust "bumper" in the event of a collision.
4. Allows the installation of the bow thrusters at a foremost position, making it more efficient.
5. Allows a larger reserve of flotation or a larger ballast capacity forward.
6. Reduces the pitch movement.

2.2 Comparison between Ice Bow and Bulbous Bow

Conventional ice bow of an icebreaker generally has inescapable disadvantage that the resistance in open sea is rather high compared with a ship having a bulbous bow. In order to solve this problem, Double Acting Tanker should has ability running astern as an icebreaker in ice bound by changing the stern part due to ice strengthening for ice breaking and pod propulsion systems and ahead in open water by introducing bulbous bow at the stem part as shown in Figure 2. The Double Acting Tanker has a bulbous bow at the stem part of ship to reduce the drag force that it was an effect of wave making resistance while ship moving ahead in open water. Therefore, the resistance of DAT will reduce that can make increasing speed and improve stability at open water.

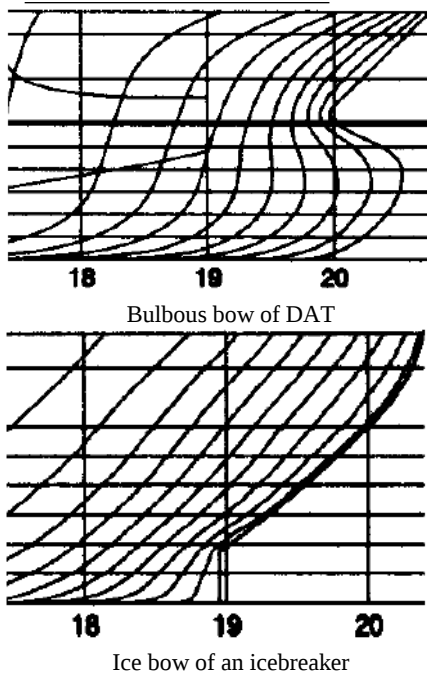


Figure 2: Difference bow shapes between common ice bow and bulbous bow [28]

2.2. Governance Equation of FEM

Once the contact zones are spotted, the local crushing force for each zone is then calculated based on the model of average contact pressure [29]:

$$F = P_H A_{cr} \quad (1)$$

Where; F is the local crushing force which is idealized as the product of the average contact pressure (P_H) and the contact area (A_{cr}).

Equation of State (EoS) as shown in Equation.1 is an equation that represents the presence of a fluid in the form of pressure and density ratios. If attention is addressed to pressure after a collision, this will become more complicated. After collision pressure will be at a high value theoretically called the peak of Hugoniot pressure.

$$p_H = \rho_0 U_S (U_0) U_0 \quad (2)$$

Where;

- p_H Hugoniot pressure
- ρ_0 material density
- U_S shock velocity
- U_0 impact velocity

After reaching the peak, pressure will decrease and the end is the stage of steady flow pressure which can be calculated using Equation 3.

$$p = \frac{1}{2} \rho_0 U_0^2 \quad (3)$$

Pressure at constant stages is easy to predict while Hugoniot pressure is also affected by shock velocity, and that is function by impact velocity too. If observed equations 1 and 2, it can be seen that pressure involved is only affected by initial density, impact and shock velocity while the impact mass unaffected by the pressure.

In this interaction review of ships with ice, ice is modeled according to linear equation of Mie-Grüneisen [30]. This equation is also known as Us-Up equation. This Mie-Grüneisen linear equation shows a linear relationship between shock and particle velocity as shown in Equation 4.

$$U_S = c_0 + s U_P \quad (4)$$

Where,

- c_0 speed of sound in material
- s material constant
- U_P particle velocity

So finally the relationship between pressure and density can be arranged like Equation 5.

$$p = \frac{\rho_0 c_0^2 \eta}{(1-s\eta)^2} \left(1 - \frac{\Gamma_0 \eta}{2}\right) + \Gamma_0 \rho_0 E_m \quad (5)$$

Where,

- $\eta = 1 - \rho_0 / \rho$ is a volumetric compressive strain
- Γ_0 material constant
- E_m internal energy in unit mass

The Mie-Grüneisen equation requires value of EOS material, and Abaqus needs ρ_0 , c_0 , Γ_0 and s . In this study, the domain is sea water so the value of $\rho_0 = 1000$, $c_0 = 1490$, $\Gamma_0 = 1.65$ and $s = 1.79$, respectively [30]

In empirically, the ice resistance is acting on the ship which can be defined below as the Equation (6):

$$F_{ice} = c \cdot \sigma \cdot h^2 \cdot \mu \cdot f(a, b) \quad (6)$$

Where:

- c is icebreaking coefficient
- σ is ice flexural strength
- h is ice thickness
- μ is coefficient of kinetic friction of ice and hull
- a, b are component angle fore or aft parts

2.3 Buckling of Ice Sheet

Buckling is characterized by a sudden side deflection of structural members. It is assumed that a ship is placed away from ice or does not come into direct contact with ice. The goal is that ship has enough energy breaking the ice. The Figure 3 shows the position of the ship at time at 0 second which is 1m in front of ice. The movement of the ship will gradually give load to the ice

sheet, along with that, the ice sheet will react in proportion to the load of the ship. These two opposite loads are concentrated around the bulbous bow. Thus, the ice sheet will slowly buckle.

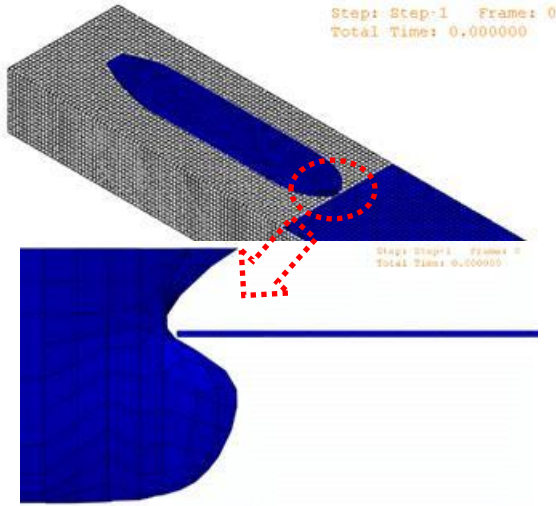


Figure 3: Placement of ship at the 1.6m distance from ice.

In order to explain the interaction between ice and hull, we consider a strut AB with length L in which the strut is applied by a compressive load, acting through its cross-sectional centroid as shown in Figure 4.

As the applied load (P) given bulbous bow increases in the structure, it will eventually become large enough to cause the structure to become unstable and curved before its elastic limit is reached.

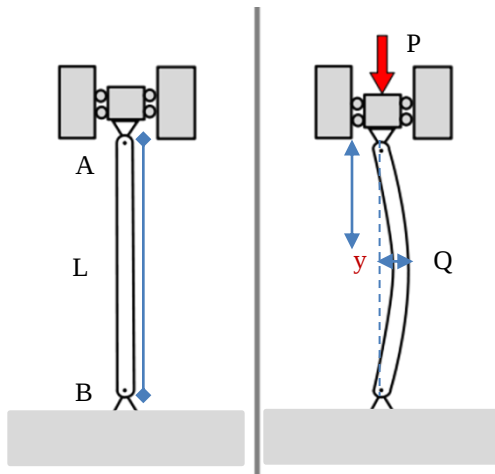


Figure 4 A strut under a concentric axial load exhibiting the characteristic deformation of buckling [32].

If Q represents a section on the elastic curve of the strut and a distance x from point A, and having transverse deflection y than bending moment at section Q of the strut is

$$M = -Py \quad (7)$$

Based on the *General Deflection Equation* for a beam,

$$M = EI \frac{d^2y}{dx^2} \quad (8)$$

Therefore

$$EI \frac{d^2y}{dx^2} = -Py$$

or

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0 \quad (9)$$

Equation 9 is a second order linear and homogeneous differential equation,

$$\text{If } \alpha^2 = \frac{P}{EI} \text{ or } \alpha = \left(\frac{P}{EI}\right)^{1/2}$$

By substituting this expression into Equation 9 results in

$$\frac{d^2y}{dx^2} + \alpha^2 y = 0 \quad (10)$$

Equation 10 is in the form of simple harmonic differential equation. The general solution for Equation 4 can be expressed in a general statement as

$$y = A \sin(\alpha x) + B \cos(\alpha x) \quad (11)$$

Where; A and B are two constants which can be determined if the boundary conditions of the strut system are known. In this case, the two boundary conditions are

$$\begin{cases} \text{if } x = 0, y = 0 \\ \text{if } x = L, y = 0 \end{cases}$$

If the boundary conditions are substituted into Equation 11 then

$$B = 0, \text{ and } A \sin(\alpha L) = 0 \quad (12)$$

$$A \neq 0, \text{ hence, } \sin(\alpha L) = 0$$

$$\text{or } \alpha L = n\pi \text{ where } n = 0, 1, 2, 3, 4, \dots$$

$$\text{Then } \alpha = \frac{n\pi}{L} \quad (13)$$

From the Equation 3

$$P = \alpha^2 EI = \left[\frac{n\pi}{L}\right]^2 EI \quad (14)$$

The smallest value of this critical load is obtained if $n = 1$, that is

$$P = \frac{\pi^2 EI}{L^2} \quad (15)$$

With; E is Modulus Young of the material, I is the smallest second moment of area of strut cross section and L is length of strut.

Equation 15 is known as Euler Equation. P_{cr} is Euler critical load. If $P \geq P_{cr}$, then buckling or elastic failure will occur.

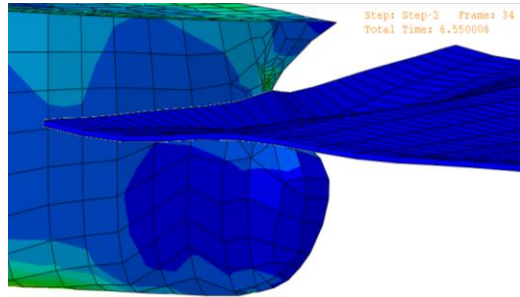
Critical buckling stress is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 EAK^2}{AL^2} = \frac{\pi^2 Ek^2}{L^2} = \frac{\pi^2 E}{(L/k)^2} \quad (16)$$

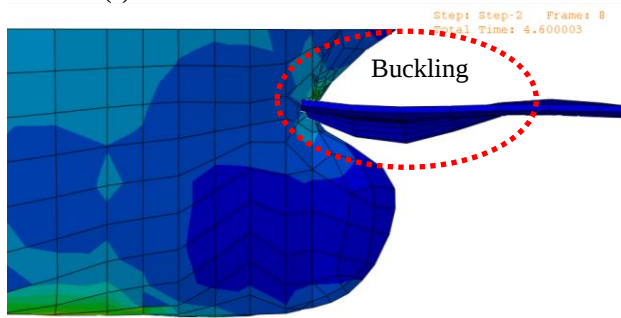
Where;

- A is cross-sectional area of strut
- k is smallest radius of gyration.
- L/k is called Slenderness Ratio of the strut.

In the study, FEM is used to analyze buckling phenomena. The sailing ahead simulation is run at several speeds of 0.4m/s, 0.5m/s and 0.6m/s. Figure.4 is taken when ship has sailed ahead head on 0.5m ice thickness with 0.5m/s speed. It is very clear to see that the ice sheet is buckled before failure.



Interaction ship into ice after travelling in 0.5m/s sailing ahead front view (a)



From side view (b)

Figure 4: Buckling of ice sheet

2.2 Deflection of Ice Sheet

Before running simulation by FEM, several important points should be arranged first. This includes preprocessing stages such as placing position ship in a fluid domain, generation of mesh, Applying Boundary Conditions, Interaction Properties, Loads,

Euler volume fraction, Predefined Fields, steps and other things which will be explained further. The stages were starting with setting position of the ship in the fluid domain. Ship is placed at a distance 1.6 m in front of ice, as shown in Figure 5. So ship will interact with water firstly before interacting with ice.

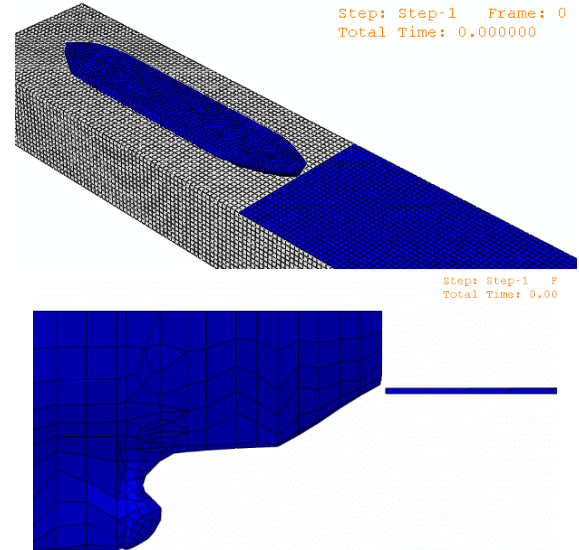


Figure 5: Setting up position of ship as stages of preprocessor stages 1.6m in front of ice.

In order to describe the deflection of ice, it is proper way to take the cantilever theorem. Figure.6 depicts a cantilever beam fixed to a vertical plane.

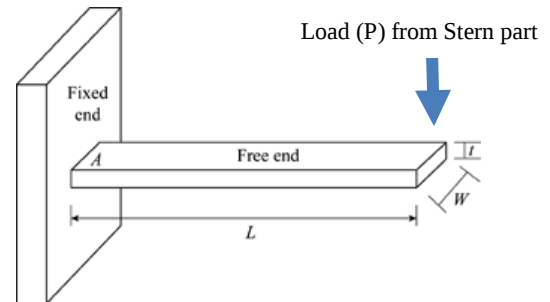


Figure 6: Cantilever beam [32]

The expressions for reactions, deflections and maximum moment for a cantilever beam of length (L) that is acted by various types of loads can be described as follows:

1. Point load P (N) at free end.

Reaction force (N) at A : P :

Maximum bending moment (Nm): $M_A = -Pl$

Maximum deflection/displacement (m), $\delta_{max} = \frac{Pl^3}{3EI}$

Slope at free end, $\theta = \frac{Pl^2}{2EI}$

2. Uniformly distributed load with magnitude P over the entire beam length

$$\text{Reaction force at A (N): } R_A = P$$

$$\text{Maximum bending moment (Nm): } M_A = -\frac{Pl^2}{2}$$

$$\text{Maximum deflection/displacement (m): } \delta_{max} = \frac{Pl^4}{8EI}$$

$$\text{Slope at free end: } \theta = \frac{pl^3}{6EI}$$

Where: E is the elastic modulus and I is second moment of inertia for beam's cross section which can be written as $= \frac{bt^3}{12}$.

The uncertainty of the second moment can be expressed as:

$$\Delta I = I \left(\frac{\Delta b}{b} + 3 \frac{\Delta t}{t} \right) \text{ using simple average method and}$$

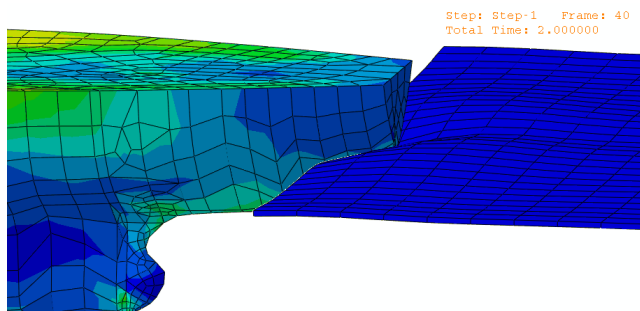
$$\Delta I = I \sqrt{\left(\frac{\Delta b}{b} \right)^2 + \left(3 \frac{\Delta t}{t} \right)^2} \text{ using standard deviation.}$$

The uncertainty of the elastic modulus can be expressed as:

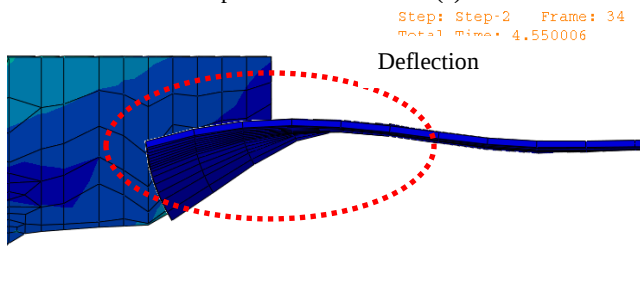
$$\Delta E = E \left(3 \frac{\Delta L}{L} + \frac{\Delta I}{I} + \frac{\Delta y}{y} \right) \text{ using simple average method and}$$

$$\Delta E = E \sqrt{\left(3 \frac{\Delta L}{L} \right)^2 + \left(\frac{\Delta I}{I} \right)^2 + \left(\frac{\Delta y}{y} \right)^2} \text{ using standard deviation.}$$

The following picture is taken when ship has sailed during 0.6s or has traveled in distance 1.08m in step-1, as shown in Figure 7.a. Right part of picture seen step-1 is completely simulated in accordance to the setup given. In this study, simulation of ice ship FEM was run at speed of 1.8 m/s, 2.4 m/s and 2.7 m/s.



Step-1 simulation at 0.6s (a)



Deflection

Step-1 simulation completed (b)

Figure 7: Deflection of ice sheet.

5.0 CONCLUSION

In conclusion, this paper discusses phenomena of buckling and deflection caused by a ship sailing in an ice level. The ice sheet buckling and deflection is analyzed based on Finite Element Method using Euler and Cantilever methods. The methods were described from general deflection equation for a beam. Simulation was carried out using Abaqus running ahead and astern at several speeds of 0.4m/s, 0.5m/s and 0.6m/s.

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