Application of Quasi-Continuous Vortex Lattice Method to Determine Aerodynamic Characteristics of Helicopter Tail Rotor Propeller

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ABSTRACT

Application of Quasi-Continuous Vortex Lattice Method (QCVLM) was widely used to predict the performance of propeller in open water and behind the hull of ships. In other hand, the application the QCVLM in aeronautics is less than in marine. This paper reviews on application of quasi continuous vortex lattice method for determining the performance of helicopter tail rotor propeller. Tail rotor blade for Bell B206 of one seat helicopter is used. The prototype helicopter was manufactured in the Aeronautics Laboratory, Universiti Teknologi Malaysia.

KEY WORDS: *Helicopter Tail Rotor Propeller; Quasi Continuous Vortex Lattice Method; Bell B206.*

NOMENCLATURE

VLM	Vortex Lattice Method
MFM	Mode Function Method
QCM	Quasi Continuous Method
SSPM	Simple Surface Panel Method
CFD	Computational Fluid Dynamic

1.0 INTRODUCTION

The propeller is the device that mainly used as propulsive for marine vehicles, airplanes and rotorcraft. As it is a crucial part, it has to be designed to meet power requirement at the indicated speed with optimum efficiency. Now days, with growing demands for of higher speed and greater power, the propeller is becoming increasingly larger in size and its geometry shape become more complicated. Due this complicated geometry, the propeller should be optimally designed for increased propulsion efficiency.

To predict the steady and unsteady propeller characteristics, many numerical models and propeller theories were proposed. One of them and will be used in this study is based on lifting surface theory. The lifting surface theory also plays as important role in the hydrodynamics analysis of marine propeller. The theory has been developed for a long time in the field of aeronautics. While almost all of the applications of the theory are to wings of airplanes, there is an old application to screw propellers (Kondo, 1942).

A number of methods based on lifting surface theory to estimate the propeller characteristics have been published. They can be classified into two groups. One group is based on the continuous loading method such as Mode Function Method (MFM) and the other the discrete loading method such as Vortex Lattice Method (VLM). Due to the complexity in numerical calculation using MFM, its application to unconventional propellers is not easy. VLM, however, also has some room to be improved; in the neighborhood of leading edge, vortex strength predicted by VLM is always lower compared with analytical solutions. Owing to the discrete and concentrated loading distribution, pressure distribution is not estimated straightforwardly. Leading edge suction force is not estimated straightforwardly either. A large number of elements are necessary to get converged solutions (Naoto Nakamura, 1985).

Taking the above circumstances into consideration, a numerical method to estimate aerodynamics characteristics of helicopter tail propeller based on Quasi-Continuous Method (QCM) will be developed. QCM has both advantages of continuous loading method and discrete loading method; loading distribution is assumed to be continuous in chord-wise direction and stepwise constant in span-wise direction. Simplicity and flexibility of discrete loading method are also retained.

2.0 LITERATURE REVIEW

2.1 Quasi Continuous Method Theory

C. Edward LAN (1974) has developed a quasi-continuous method for solving the thin wing problem. In his study, the spanwise vortex distribution is assumed to be stepwise constant while the chordwise vortex integral is reduced to a finite sum through a modified trapezoidal rule and the theory of Chebychev polynomials, in order to satisfying the boundary condition. Wing edge and Cauchy singularities are accounted for, based on two-dimensional theory; that contain the conventional Vortex Lattice Method and the present analysis. Based on his conclusions, the wing square-root singularities and the Cauchy singularity has been properly accounted for in this method, compared to the conventional vortex lattice method. The total aerodynamic characteristics are obtained by an appropriate quadrature integration. The two-dimensional results for airfoil without flap deflection reproduce the exact solutions in lift and pitching moment coefficients, the leading edge suction, and the pressure difference at a finite number of points. For a flapped airfoil, the present results are more accurate than those given by the vortex lattice method. The three dimensional results also show an improvement over the results of the vortex lattice method.

Naoto Nakamura (1985) has conducted research on estimation of propeller open- water characteristics base on Quasi – Continuous Method. This study show that comparisons of propeller open-water calculated by this method with measured data showed good agreement for a wide variety of propellers. This method is promising for the improvement of numerical methods in propeller lifting surface theory. However, for propeller with extremely large skew, the estimation accuracy was not so good. Nakamura recommended for improvement, a more realistic wake model should be formulated based on the geometry of slipstream such as contraction, variation of pitch etc. Force acting on source distribution and leading edge suction force should be properly accounted for. And lastly, more exact treatment of blade thickness is necessary for the improvement in the calculation accuracy of pressure distribution on blade surface.

J.Ando and KuniharuNakatake (1999) have conducted a calculation method for the three – dimensional unsteady sheet cavitation hydrofoil problem. This method is based on simple surface panel method "SQCM" which satisfied easily the Kutta condition even in the unsteady condition problem. This method uses source distribution (Hess and Smith, 1964) on the hydrofoil surface and discrete vortex distribution arranged on the mean chamber surface according to QCM (Quasi-Continuous vortex lattice Method) (Lan, 1974). These singularities should satisfied the boundary condition that the normal velocity is zero on the hydrofoil and the mean camber surface. The partially cavitation hydrofoil was treated in heaving motion and both partially cavitation and super cavitation was in sinusoidal gust. The calculated results ware compared with other calculated results and the accuracy of the present result is confirmed.

J. Ando and K. Nakatake (2000) have conducted research on a new surface panel method to predict steady and unsteady characteristic of marine propeller. These methods represent the flow field around a wing by the source and the doublet distributions on the wing surface, and are applied successfully to calculation of the pressure distribution on the propeller blade. But it is not easy to satisfy the Kutta condition at the tailing edge and many discussions were raised about it (Koyama, 1989). A reasonable expression of the Kutta condition is that the pressure becomes equal on the upper and lower wing surfaces at the tailing edge. In order to satisfy the above condition, it is need to solve iteratively the quadratics simultaneous equations for singularity distributions. They have developed a new surface panel method which is a kind of singularity method. This method uses source distributions (Hess and Smith, 1964) on the wing surface and discrete vortex distribution arranged on the chamber surface according to quasi-continuous method vortex lattice method (QCM) (Lan,1974). Then these singularities both the wing surface and the camber surface condition that the normal velocity should be zero in case of the steady problem. Since these singularities satisfy automatically Kutta condition, they not need use any iterative procedure. In case of the unsteady problem, they introduce the normal velocity to the chamber surface at the trailing edge in order to make the pressure difference between the upper and the lower wing surface zero. Even though the iterative procedure to satisfy the Kutta condition is necessary, the convergence is quite fast. They named this simple surface panel method SQCM (source and QCM). In SQCM, the upper and the lower surface of the propeller blades are divided into the same numbers of source panels, respectively. Then the ring vortices are located on the mean chamber according to the unsteady QCM (Hoshino, 1985) (Murakami et al., 1992). The hub surface is also divide into source panels. The trailing vortices flow out as ring vortices from the trailing edge every time step. They assume that the trailing vortices leave the trailing edge in tangential to the mean chamber surface and the pitch of trailing edge vortices reach an ultimate value, which is the mean of geometrical pitch distribution of the propeller blade, within a haft revolution. They solved the boundary conditions every time step Δt. The source distributions on propeller blades and hub surface and the vortex distributions on the chamber surface are obtained for each time step from the boundary conditions at the center of the source panels and control points on chamber surfaces whose positions are determined by the QCM theory. Here the pressure Kutta condition is considered simultaneously. The perturbation potentials and velocity distributions on the blade surfaces are obtained from the singularity distributions. The pressure distributions are calculated by the unsteady Bernoulli's equation. Then the force acting on propeller blades are obtained by pressure integration on the propeller blade.

J.Ando (2009) has done the numerical analysis of steady and unsteady sheet cavitation on a marine propeller using a Simple Surface Panel Method (SSPM). This SSSPM uses source distribution on propeller blade surface from Hess & Smith (1964) and discrete vortex distributions arranged on the camber surface according to QCM (Quasi-Continuous vortex lattice Method) by Lan (1974). The singularities satisfy the boundary condition that the normal velocity is zero on the propeller blade and mean chamber surface. From this analysis Jun Ando concluded that present method can predict the thrust and the torque in extensive advance coefficients and cavitation numbers. For unsteady case, this method can express the variations of the cavity area and cavity volume. And the cross flow effect on the cavity surface gives reasonable cavity shapes and variations of cavity area and volume especially in HSP.

2.2 Lifting Surface Theory

Research on application of the lifting surface theory to marine propeller was carried out by Koichi Koyama (2012). In this research they use three methods, named here Method 1, Method 2, and Method 3, of propeller lifting surface theory proposed by Hanoaka (1969). Method 1 and Method 3 in this research belong to mode function method group, and Method 2 belongs to discrete function method group. Method 1 may be called series expansion method. Two – dimensional integral equation for lifting surface is converted into simultaneous one-dimensional integral equations, in expanding the two-variable integral equation in power series of one- variable by Taylor' theorem and equating the coefficient functions of successive powers to zero. The method is equivalent to Flax's method on condition that the mode function is orthogonal series. The method, however, is distinguished by easy calculation of coefficients of simultaneous equations in comparison with Flax's method. The method can deal with the simple mean chamber line accurately without many chordwise control points in contrast to the collocation method. The method reduces the labour for numerical integration along the helical surface to the minimum. Especially for the case of unsteady condition, the machine running time for calculation not long. In Method 2, existing integral equation is solved by the doublet-lattice method. The blade is divide into many blade element by cylindrical surface, and each blade element is divided into many boxes. Surface distribution of doublet in the pressure field is replaced by line doublets with constant strength and direction in every box. The chordwise arrangement is determined by Lan's method, which is devised so as to give the accurate solution near the leading edge. Method 1 yields valid circulation density in wide range near the tip and yields valid circulation of blade section in all range for the usual broad bladed propeller. The method is useful for the calculation of propeller forces. Method 2 gives the converged chordwise distribution of circulation density. It is supposed to give the accurate solution near the leading edge and to be useful for the calculation cavitation. Method 3 yield the accurate solutions for the circulation density near the blade tip as well, even though the blades are highly broad. The method is considered useful for the calculation of cavitation. There are many possibilities of application of the propeller lifting surface theory to the propeller design. The following are some examples of the application method to be employed in the design procedure. The blade outline is adjusted according to the inflow variation of hull wake from the view point of vibratory propeller forces, when Method 1 is useful. If the blade outline is given, the relation between upwash and loading is controlled by the lifting surface theory. If the pitch of the helical surface is assumed to fix, single calculation of the matrix of coefficient corresponding to the integral equation, make it possible to obtain upwash distribution and to obtain loading distribution by the upwash distribution with ease. In this case, Method 2 and Method 3can control accurately the pitch correction or chamber correction in the design method, from the cavitation point of view.

Koichi Koyama (2012) has conducted relation between the lifting surface theory and the lifting line theory in the design of an optimum screw propeller. This research is based on the propeller lifting surface theory. Circulation density (lift density) of the blade is determined by the lifting surface theory on a specified condition in general. However, it is shown that, in the case of optimum condition, the circulation density is not determined by the lifting surface theory, although the circulation distribution which is the chordwise integral of the circulation density is determined. The reason is that the governing equation of the optimization by the lifting surface theory is reduced to that by the lifting line theory.

2.3 Propeller

Davide Grassi has done numerical for analysis of the dynamics characteristics propeller in uniform and stationary inflow by lifting surface theory where this lifting surface analysis method based upon Kerwin (1978) theory. These procedures employs a simplified defined shape for representing propeller wake geometry which given good results for moderately skewed blade geometries not too far from their design point, while tend to overestimate thrust and torque at low advance coefficient. To overcome this difficulties, DavideGrassi was employed this analysis fully numerical wake adaption method. The present tool employs a discrete vortex line representation of the blade, with elements located at the exact mean surface. The leading edge suction and tip vortex separation was consider in an accurate and efficient way. The propeller was assumed to work in an inflow with radially varying axial and tangential component, while the presents of the hull hub and free surface is discounted. The blade force calculation was separated into three components, force acting on vortex sheet elements that determined by using Kutta – Joukowski theorem, force acting on source singularity determined by using Lagally's theorem, and viscous drag forces are calculated at each radial position using a global frictional coefficient experimentally. Tip separation effect has been considered based on the Kerwin's formulation. These segment with the local velocity vector is necessary to align, since chordwise vortex element are to be force free. Vortex ring corner points placed at bladed tip are all shifted in a direction normal to the mean surface by a quality linearly proportional to its position on the tip chord. The effect of tip vortex separation is in the logically to increase propeller thrust and torque at low advance coefficient, while its effect is negligible around the design advance coefficient.

A. Brocklehurst (2012) has make the review of helicopter tip shapes to discussed the evolution of helicopter blade designs and tip shapes, and the development of methods for their analysis and evaluation. The review has identified three main types of helicopter tip designs: the parabolic tip, the swept (tapered) tip, and the BERP tip. In addition there are several tip shapes intended to alleviate BVI noise by splitting, or diffusing the tip vortex. The literature review has identified some overlap with fixed-wing aircraft, particularly with regard to tip aerodynamics and the details of vortex formation. While there are some obvious fundamental differences, both fixed-wing and rotary-wing configurations require accurate resolution of the flow-field if the results are to be useful in tip design studies. The nature of the helicopter problem is, however, much more complex than fixed-wing and is much more demanding from a modelling point of view due to the need to preserve wake vortices, take into account variations in incidence and sideslip (in cruising flight), and also include unsteady effects. The design must also work within much tighter moment constraints. Nevertheless, much can be learnt from validating the methods against fixed-wing tests, and there is considerable carry-over in some of the detailed design thinking. In particular, the work of Hoerner and K¨uchemann has provided a basis for generating improved tip shapes. The review has also covered the development of computational methods. Analysis techniques have developed from rotor models based on lifting-line and lifting-surface theory, with a prescribed or free wake, to the application of sophisticated CFD methods. Initially the numerical approach required some of the world's largest computers, but these modern methods are now becoming available in industry, and offer greater insight and higher resolution than traditional design methods. Two main features of the computational approach would appear to require further development for helicopter applications. One is that it is important to fully capture the wake in order to determine the correct flow-field around the rotor blade, and the other is that the solver needs to include boundary layer transition in both steady and unsteady (3D) conditions, if accurate overall performance and the effects of retreating blade stall are to be predicted. Much effort has been expended on coupling wake models with near-field Navier–Stokes solutions in an effort to obtain the desired accuracy at acceptable computational cost. It is also clear that rotor trim and blade deflections are important aspects of helicopter simulation. Often, the rotating and pitching blades may operate in proximity to a stationary fuselage, or fin, and this presents challenges in grid generation, requiring the use of sliding grids, or Chimera overset grid techniques. From the literature there appears to be a preference toward structured grids to resolve the flow features, but some developers favor the Chimera approach. It is also clear that CFD methods are maturing quickly and through rapidly growing computer resources will soon be universally accepted and indispensable for rotor blade design evaluations. Recent research effort on CFD has been directed towards tackling the challenging problems of whole helicopter simulations. Coupling of CFD with structural dynamics to take into account blade deflections has also been a focus of attention, and offers to greatly improve the fidelity of the simulation. Most recently, fast harmonic-balance methods have begun to emerge which promise further run-time reductions for forward flight simulations. However, while CFD has been applied to a range of rotors problems, including investigations of tilt-rotors, little new work has been found on the use of CFD for the design of new tip shapes, particularly for tail rotors.

3.0 UTM ONE SEATED HELICOPTER

One of the very first problems helicopter designers encountered when they tried to create a machine that could hover was the problem of torque reaction (Guo-HuaXu, 2001). Newton's third law of motion requires that for every action there is an equal and opposite action. A typical single main rotor helicopter has a rotor system mounted on a rotor mast. The helicopter engine supplies power so that the helicopter can turn the mast, and thus the rotor system connected to it. When the helicopter applies torque to the mast to spin it, there is an equal-and-opposite torque reaction which tries to turn the helicopter in the opposite direction as shown in Figure 1.

Igor Sikorsky seems to be the first to settle on using a single rotor mounted at the rear of the helicopter as a way to counter the torque. This is the most popular arrangement today. Sikorsky actually experimented with many different arrangements before selecting a single tail mounted rotor. It seems strange that the majority of helicopters produced use this method of countering torque, given that there are several major problems with this method which are not encountered with counter-rotating rotor systems (J. Gordon Leishman, 2002).

One major problem with tail rotors is that they rob an enormous amount of power. As a rule of thumb, tail rotors consume up to 30% of the engine power (C. Young, 1976).Still another problem with tail rotors is that they are fairly difficult to control accurately. Turbulence and crosswinds make it extremely difficult to hold a constant heading in a tail rotor equipped helicopter. The workload is very high, and good results are difficult to achieve. Many larger helicopters end up being designed with a yaw stabilization system, which is essentially an autopilot for the tail rotor.In the study, one seat helicopter is used as a case study as shown in Figure.2

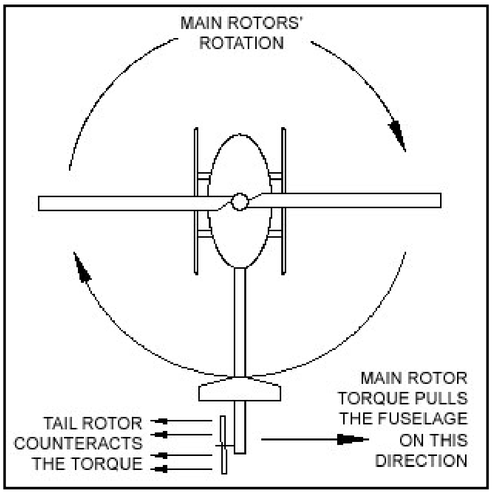

Figure 1: Tail rotor function

Figure.2: One seated helicopter.

3.1 Tail Helicopter Rotor Blade

In this study, offsets coordinate from tail rotor blade for Bell B206 helicopterwill be used as simulation data to calculatethe air forces and other performance characteristics on the rotor blade. Experimental will be done to make comparison with the data from simulation to ensure that data will be valid for both simulation and experimental.

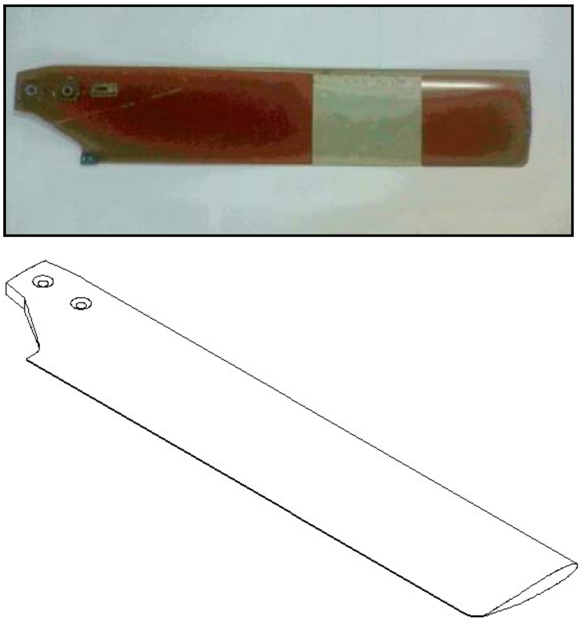

Figure 3: The real model of tail rotor blade for Bell B206.

4.0 QUASI-CONTINUOUS VORTEX LATTICE METHOD

4.1 Outline of QCM

The propeller blade is divided into M panels in the spanwise direction. The face and back surfaces of the blade section are divide into N panels in the chordwise direction, respectively. Therefore, the total number of source panels becomes*(M × 2N)* × *K* and constant m is distributed in each panel. The total *(M × 2N)* × *K* horse shoe vortices are located on the mean chamber surface according to Eq. 1 as illustrated in Figure 5.

$$\xi^{LP}_{\mu\nu} = \xi_L(r_\mu) + \frac{\xi_T(r_\mu)-\xi_L(r_\mu)}{2}\left(1 - \cos\frac{2\nu-1}{2N_\gamma}\pi\right) \qquad (1)$$

$$\xi^{CP}_{\mu\nu} = \xi_L(\bar{r}_\mu) + \frac{\xi_T(\bar{r}_\mu - \xi_L(\bar{r}_\mu)}{2}\left(1 - \cos\frac{\nu}{2N_\gamma}\pi\right)$$

where,

$$\bar{r}_\mu = \frac{1}{2}(r_\mu + r_{\mu+1}) \qquad (2)$$

μ and ν are numbers in the spanwise and chordwise directions.

$\xi_L(r_\mu)$ and $\xi_T(r_\mu)$ are thepositions of the leading edge (LE) and trailing edge (TE), respectively.

The induced velocity vector due to ν-th ring vortex of unit strength which start from the μ-th strip and ones due to the ℓ-th ring vortex of unit strength located in trailing wake defined as $\vec{v}^{\gamma}_{k\mu\nu}$ and $\vec{v}^{w}_{k\mu\ell}$. These induced velocity vectors are expressed as

$$\vec{v}^{\gamma}_{k\mu\nu} = \vec{v}^{B}_{k\mu\nu} + \sum_{\acute{\nu}=\nu}^{N_\gamma}\left(\vec{v}^{F}_{k\mu+1\acute{\nu}} - \vec{v}^{F}_{k\mu\acute{\nu}}\right) - \vec{v}^{ws}_{k\mu\ell}|_{\ell=1} \quad (3)$$

$$\vec{v}^{w}_{k\mu\ell} = \vec{v}^{ws}_{k\mu\ell} - \vec{v}^{ws}_{k\mu\ell+1} + \vec{v}^{wc}_{k\mu+1\ell} - \vec{v}^{wc}_{k\mu\ell}$$

where, $\vec{v}^{B}_{k\mu\nu}$ is the induced velocity vector due to the bound vortex of unit strength on the mean camber surface, $\vec{v}^{F}_{k\mu\acute{\nu}}$ the induced velocity vector due to the free vortex of unit strength on the mean camber surface, $\vec{v}^{ws}_{k\mu\ell}$ the induced velocity vector due to the spanwise shed vortex of unit strength in the trailing wake and $\vec{v}^{wc}_{k\mu\ell}$ is the induced velocity vector due to the streamwise trailing vortex of unit strength in the trailing edge.

The induced velocity vector $\vec{V}$ due to each line segment of vortex is calculated by the Biot-Savart law. The segment of the ring vortex on the mean chamber surface at time t_L and the ring vortex in the trailing edge wake defined as $\gamma_{k\mu\nu}$ (t_L) and $\Gamma_{k\mu}$ (t_L), the induced velocity vector due to the vortex model of the QCM theory in given by the following equation.

$$\vec{V} = \sum_{k=1}^{K}\sum_{\mu=1}^{M}\sum_{\nu=1}^{N_\gamma}\gamma_{k\mu\nu}(t_L)\vec{v}^{\gamma}_{k\mu\nu}\Delta\xi_L + \sum_{k=1}^{K}\sum_{\mu=1}^{M}\sum_{\ell=1}^{L-1}\Gamma_{k\mu}(t_{L-\ell})\vec{v}^{\gamma}_{k\mu\nu}\Delta\xi_L\vec{v}^{w}_{k\mu\ell} \quad (4)$$

where

$$\Delta\xi_{\mu\nu} = \frac{\pi c(\bar{r}_\mu)}{2N_\gamma}\sin\frac{2\nu-1}{2N_\gamma}\pi, \qquad \Gamma_{k\mu}(t_L) = \sum_{\nu=1}^{N_\gamma}\gamma_{k\mu\nu}(t_L)\Delta\xi_{\mu\nu},$$

and $c(\bar{r}_\mu)$ is the chord length of μ section.

This Equation (4) is used when the control points are on the mean camber surface (J. Ando, 2012). When the control points are on the blade surface, the ring vortices are close to the control points especially for the thin wing. For this case, the ring vortices on the mean camber surface and shed vortex nearest to T.E. treat as the vortex surface in order to avoid numerical error. This is called the "thin wing treatment" (S. Maita, 2000).

J. Ando and T. Kanemaru (2012) has been modify the vortex model of the QCM theory (Fig. 5b) as follows:

For first step, the ring vortex on the mean camber surface at T.E. is close and locate the ring vortices ABCH, HCDG and GDEF downstream from T.E. as shown in (Fig. 5b).

After that the ν-th ring vortex replace with a set of ring vortices distribution along $\Delta\xi_{\mu\nu}(\xi^{CP}_{\mu\nu-1}\sim\xi^{CP}_{\mu\nu})$ on the mean camber surface as illustrated in the middle of Fig. 5b. Then the ring vortex ABCH just downstream from T.E. and ring vortex HCDG are replaced with a set of ring vortices distribution along $\Delta\xi^{w}_{\mu}$ in the trailing wake. The other ring vortices form $\Delta\xi^{w}_{\mu}$ are calculated as discrete ring vortices.

Figure 4: Tail propeller blade of helicopter.

For this case, the induced velocity vectors due to the vortex systems on the mean camber surface and trailing wake are expressed as

$$\vec{V_\gamma} = \sum_{k=1}^{K}\sum_{\mu=1}^{M}\sum_{\nu=1}^{N_\gamma}\gamma_{k\mu\nu}(t_L)\int_{\xi^{CP}_{k\mu\nu-1}}^{\xi^{CP}_{k\mu\nu}}\vec{v}^{\gamma}_{k\mu}(\xi)d\xi + \sum_{k=1}^{K}\sum_{\mu=1}^{M}\sum_{\nu=1}^{N_\gamma}\gamma_{k\mu\nu}(t_L)\frac{\Delta\xi_{\mu\nu}}{\Delta\xi^{w}_{\mu}}\int_{\xi_T(\bar{r}_\mu)}^{\xi_T(\bar{r}_\mu)+\Delta^{w}_{\mu}}\vec{v}^{ABCH}_{k\nu}(\xi)d\xi + \sum_{k=1}^{K}\sum_{\mu=1}^{M}\Gamma_{k\mu}(t_L-1)\frac{1}{\Delta\xi^{w}_{\mu}}\int_{\xi_T(\bar{r}_\mu)}^{\xi_T(\bar{r}_\mu)+\Delta^{w}_{\mu}}\vec{v}^{HCDG}_{k\mu}(\xi)d\xi$$
$$+ \sum_{k=1}^{K}\sum_{\mu=1}^{M}\Gamma_{k\mu}(t_L-1)\vec{v}^{GDEF}_{k\mu} + \sum_{k=1}^{K}\sum_{\mu=1}^{M}\sum_{\ell=2}^{L-1}\Gamma_{k\mu}(t_L-\ell)\vec{v}^{w}_{k\mu\ell} \quad (5)$$

where

$$\vec{v}^{\gamma}_{k\mu\nu}(\xi) = \vec{v}^{B}_{k\mu}(\xi) + \left(\vec{v}^{F}_{k\mu+1}(\xi) - \vec{v}^{F}_{k\mu}(\xi)\right) + \sum_{\nu'=\nu+1}^{N_\gamma}\left(\vec{v}^{F}_{k\mu+1\acute{\nu}} - \vec{v}^{F}_{k\mu\acute{\nu}}\right) - \vec{v}^{AB}_{k\mu}$$

$$\vec{v}_{k\mu}^{ABCH}(\xi) = \vec{v}_{k\mu}^{AB} + \vec{v}_{k\mu+1}^{BC}(\xi) - \vec{v}_{k\mu}^{AH}(\xi) - \vec{v}_{k\mu}^{HC}(\xi)$$
$$\vec{v}_{k\mu}^{HCDG}(\xi) = \vec{v}_{k\mu}^{HC}(\xi) + \vec{v}_{k\mu+1}^{CD}(\xi) - \vec{v}_{k\mu}^{HG}(\xi) - \vec{v}_{k\mu}^{GD}$$
$$\vec{v}_{k\mu}^{GDEF} = \vec{v}_{k\mu}^{GD} + \vec{v}_{k\mu+1}^{DE} - \vec{v}_{k\mu}^{GF} - \vec{v}_{k\mu}^{FE}$$

The velocity vector $\vec{V}$ around a propeller in the propeller-fixed coordinate system is expressed as

$$\vec{V} = \vec{V}_I + \vec{V}_\gamma + \vec{V}_m, \quad (6)$$

where $\vec{V}_I$, $\vec{V}_\gamma$ and $\vec{V}_m$ are inflow, induced velocity vectors due to vortex and source distributions, respectively.

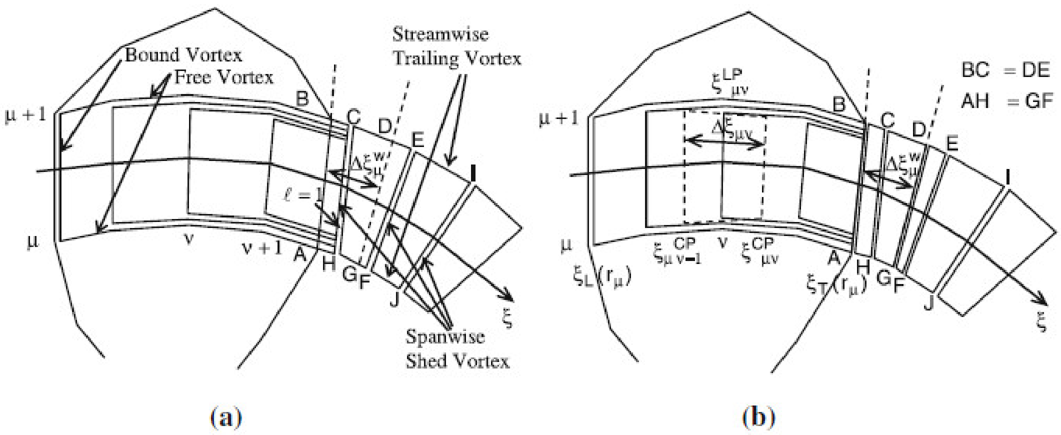

Figure 5: Arrangement of vortex system. a) From QCM. b) Thin wing treatment.

4.2 Boundary Condition and Numerical Procedure

The boundary condition of the control points on the rotor blade and mean camber are that there is no flow across the surface. But, to satisfy the Kutta condition as expressed by Equation (7), there exists at T.E the normal velocity which was introduced. So, the equation for the boundary condition as expressed by Equation (8).

$$\Delta p = \rho \frac{\partial}{\partial t}(\phi_+ - \phi_-) + \frac{1}{2}\rho(V_+^2 - V_-^2) \quad (7)$$

$V\,.n = 0$ on wing and camber surfaces (except T.E)

$$V\,.n = V_N \quad \text{at T.E} \quad (8)$$

Where *n* is the normal vector on the blade and mean camber.

The unknown value are the vortex strength on the mean camber surface, source strength on the blade and normal velocity $\vec{V}_N$ at T.E. $\vec{V}_N$ is expressed by the following equation and determined so as to satisfy the Kutta condition iteratively.

$$V_N^{(i+1)} = \frac{\Delta p^{(i)}\beta}{\rho|V_I|} + V_N^{(i)} \quad (9)$$

Where i is the number iteration. From the Equation (7), ϕ_+ and ϕ_- or V_- and V_+ are the mean of the magnitude or perturbation potential of the velocity at the control point on the upper and the lower surface blade, where it is close to T.E. This iteration is move on until Δp is small enough. According to J. Ando (2012), in this calculation $\beta = 1$ and β means the accelerations factor. In the unsteady of QCM, the Kutta condition is when $\Delta p = 0$ .The first term in the right hand side in Equation (9) means the corrector for the value of the previous step $V_N^{(i)}$ which is proportional to the pressure difference $\Delta p^{(i)}$. The iteration is continued until the pressure difference becomes small ($\Delta C_p < 0.5 \times 10^{-3}$). Assuming the normal velocity V_N at T.E. at each iterative step, Equation (8) can be solved as the linear simultaneous equations for singularity distributions.

4.3 Unsteady Pressure and Propeller Forces

The unsteady pressure distribution on the propeller blade is calculated by the unsteady Bernoulli equation expressed as

$$p(t) - p_0 = -\frac{1}{2}\rho(|V|^2 - |V_I|^2) - \rho\frac{\partial\phi}{\partial t} \quad (10)$$

Where; p_0 is the static pressure in the undisturbed inflow, p is the density of the fluid and ϕ is the perturbation potential in the propeller coordinate system The time derivative term of ϕ in Equation (10) is obtained numerically by two points upstream difference scheme with respect to time.

The blade pressure of the propeller is expressed as the pressure coefficient C_p in order to compare the calculate results with the experimental data. The pressure coefficient is defined by

$$C_p = \frac{p(t) - p_0}{\frac{1}{2}\rho n^2 D^2} \quad (11)$$

where n is the rate of the revolutions of the propeller and D is the diameter of the propeller.

The lift, drag and moment coefficients are given by the relationships

$$C_L = \frac{L}{\frac{1}{2}\rho A V^2}, C_D = \frac{D}{\frac{1}{2}\rho A V^2}, \text{ and } C_M = \frac{M}{\frac{1}{2}\rho A l V^2} \quad (12)$$

in which A is the propeller area, l is a reference length, V is the free stream incident velocity, ρ the density of the fluid, L and D are the lift and drag forces, perpendicular and parallel respectively to the incident flow, and M is the pitching moment defined about a convenient point.

For analysis purposes, the sectional lift, drag and moment coefficients are given by the relationships

$$c_L = \frac{L}{\frac{1}{2}\rho c V^2}, C_D = \frac{D}{\frac{1}{2}\rho c V^2}, \text{ and } C_M = \frac{M}{\frac{1}{2}\rho c^2 V^2} \quad (13)$$

in which c is the section chord length and L', D' and M' are the forces and moments per unit span.

The thrust T and the torque Q of the propeller are calculated by pressure integration. Denoting the x-, y- ad z- components of the normal vector on the blade surface by n_x, n_y and n_z, respectively, the thrust and the torque are expressed by

$$T(t) = \int\int_{S_B+S_H} (p(t)-p_0) n_x ds \quad , \quad Q(t) = \int\int_{S_B+S_H} (p(t)-p_0)(n_y z - n_z y) ds \quad (14)$$

According to Naoto Nakamura (1985), in the steady calculation the viscous components of the thrust and the torque are considered approximately using the viscous drag coefficient of blade element C_d at the radius r. these component T_D and Q_D are expressed as

$$T_D = -K \frac{1}{2}\rho \int_{r_H}^{r_0} C_d(r) \overline{W}(r) \overline{V_x}(r) c(r) dr$$

$$Q_D = K \frac{1}{2}\rho \int_{r_H}^{r_0} C_d(r) \overline{W}(r) \overline{V_\theta}(r) c(r) dr$$

$$C_d(r) = \begin{cases} 0.0084 + 0.016\left(\frac{t}{c}\right) & for\ C_l < 0.2 \\ 0.0084 + 0.016\left(\frac{t}{c}\right) + 0.03(C_l - 0.2) & for\ C_l \geq 0.2 \end{cases} \quad (15)$$

where

c_l = sectional lift coefficient

$\overline{W}(r) = \sqrt{\overline{V_x}(r)^2 + \overline{V_\theta}(r)^2}$

$\overline{V_x}(r)$ = x- component of resultant flow velocity averaged in the chord wise direction

$\overline{V_\theta}(r)$ = θ- component of resultant flow velocity averaged in the chord wise direction

$c(r)$ = chord length of the rotor blade

r_0 = propeller radius

r_H = hub radius

t/c = thickness ratio

The force F_X, F_Y, F_Z and moments M_X, M_Y, M_Z is acting on the propeller in the X, Y, Z directions of the space coordinate system are expressed as

$$F_X(t) = -T(t) = \int\int_{S_B+S_H} (p(t)-p_0) n_x ds$$

$$F_Y(t) = \int\int_{S_B+S_H} (p(t)-p_0)(n_y \cos(\Omega t) + n_z \sin(\Omega t))\ ds$$

$$F_Z(t) = \int\int_{S_B+S_H} (p(t)-p_0)(n_z \cos(\Omega t) - n_y \sin(\Omega t))\ ds$$

$$M_X(t) = Q(t) = -\int\int_{S_B+S_H} (p(t)-p_0)(n_y z - n_z y) ds$$

$$M_y(t) = -\int\int_{S_B+S_H} (p(t)-p_0)[(n_z x - n_x z)\cos(\Omega t) + [(n_x y - n_y x)\sin(\Omega t)] ds$$

$$M_z(t) = -\int\int_{S_B+S_H} (p(t)-p_0)[(n_x y - n_y x)\cos(\Omega t) + [(n_x z - n_z x)\sin(\Omega t)] ds \quad (15)$$

For the force and moments are non-dimensionalized as below:

$$K_{FX}(t) = \frac{F_X(t)}{\rho n^2 D^4} \quad , \quad K_{FY}(t) = \frac{F_Y(t)}{\rho n^2 D^4}$$

$$K_{FZ}(t) = \frac{F_Z(t)}{\rho n^2 D^4} \quad , \quad K_{MX}(t) = \frac{M_X(t)}{\rho n^2 D^5}$$

$$K_{MY}(t) = \frac{M_Y(t)}{\rho n^2 D^5} \quad , \quad M_{MZ}(t) = \frac{M_Z(t)}{\rho n^2 D^5} \quad (16)$$

And finally the advance J, the torque and the thrust coefficient K_Q, K_T and the propeller efficiency η_p are expressed as follows:

$$J = \frac{V_A}{nD} \quad , \quad K_T = \frac{T}{\rho n^2 D^4} \quad K_Q = \frac{Q}{\rho n^2 D^5} \quad , \quad \eta_p = \frac{J}{2\pi}\frac{K_T}{K_Q} \quad (17)$$

5.0 CONCLUSION

In conclusion, this paper reviewed on prediction of performance of helicopter tail propeller using quasi continuous vortex lattice method. As a case study, tail rotor blade for Bell B206 for one seat helicopter is used.

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