Semi-Submersible Heave Response Study Using Diffraction Potential Theory with Viscous Damping Correction

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ABSTRACT

This paper discusses the numerical prediction of the semi-submersible’s heave motion. In the previous study, it is observed that the heave motion response predicted by diffraction potential theory is over-estimated in the region where the heave motion is dominated by damping. In this research, viscous damping is included in the calculation to increase the heave damping magnitude in motion equation. The wave force and added mass of semi-submersible is predicted by diffraction potential theory, only the total damping is corrected by sum-up the linear damping from diffraction potential theory with the proposed viscous damping. The heave motion response obtained from the proposed numerical method also compared to the data from the experiment. From the comparison, it can conclude that involved of viscous damping in the calculation will corrected a part of heave motion response tendency and reduce the large over-predicted error of heave motion response at the damping dominate.

KEY WORDS: Diffraction Potential, Viscous Damping, Heave Motion, Semi-Submersible.

NOMENCLATURE

\( \Phi(x, y, z) \) \quad Wave Potential
\( A_{ij} \) \quad Added Mass
\( B_{ij} \) \quad Linear Damping
\( G(P; Q) \) \quad Green Function
\( v \) \quad Damping Ratio
\( w_n \) \quad Natural Frequency
\( E_I \) \quad Wave Force
\( \delta \) \quad Logarithm Decrement
\( w_d \) \quad Damped Natural Frequency
\( n_i \) \quad Normal Vector

1.0 INTRODUCTION

This work proposes a damping correction method for linear diffraction theory in order to evaluate the heave motion response of selected offshore floating structure correctly. The linear diffraction theory estimate the wave force on the floating body based on frequency domain and this method can be considered as an efficient method to study the motion of the large size floating structure with acceptable accuracy. This is because previous study also observed that the diffraction effect is significant for the large structure [16]. However, the offshore structure such as semi-submersible, TLP and spar are looked like a combination of several slender bodies as an example, branching for semi-submersible.

In this study, semi-submersible structure is selected as an offshore structure model because this structure is one of the flavours structures used in deep water oil and gas exploration area. To achieve this objective, a programming code was developed based on diffraction potential theory and it is written in visual basic programming language. By comparing the numerical result executed by using diffraction potential theory to experiment result, it is obtained that this theory able to predict the motion response for semi-submersible with acceptable accuracy most of the time, except for heave motion when the wave frequency near to the structure natural frequency [17].

As presented in previous paper, diffraction potential theory is less accurate to predict the structure heave motion response when the wave frequency closer to structure natural frequency. At this situation, the heave response calculated by the diffraction
potential theory will be overshooting to infinity compare to
experiment result due to low damping executed by the theory. 
However, the diffraction potential theory still able to catch 
the heave response accurately other than damping dominate region [17].

In order to correct the over-predicting phenomenon made by 
the diffraction potential theory, this research was trying to 
increase the damping coefficient by adding viscous damping into 
the motion equation. From that study, the viscous damping is 
treated as extra matrix and can be added into the motion equation 
separately. This addition viscous damping was estimated based 
on the equation proposed by S. Nallayarasu and P. Siva Prasad in 
their published paper [19].

Accuracy of this modification solution was also checked 
with the previous semi-submersible experiment result which 
carried out at towing tank belongs to University Teknologi 
Malaysia [12, 18]. The experiment is conducted in head sea 
condition and slack mooring condition for wavelength from 
1 meter to 9 meters. The comparison obtained that by adding the 
extra viscous damping into the motion equation, it can reduce 
error on the significant over-predicting of heave motion at the 
situation when wave frequency near to the semi-submersible 
natural frequency. This also caused the predicted motion response 
tendency is closed to the experimental result compared to 
executed result by diffraction potential theory alone. Besides, 
this paper will discuss the proposed viscous damping equation in the 
begining and then the effect addtion viscous damping change 
the motion equation. The difference of heave response results 
obtained from diffraction potential theory with the existing of 
viscous damping and without viscous damping is also discussed 
here.

2.0 LITERATURE REVIEW

Hess and Smith, Van Oortmerssen and Loken studied on non-
lifting potential flow calculation about arbitrary 3D objects [1, 2, 
3]. They utilized a source density distribution on the surface of 
the structure and solved for distribution necessary to lake the 
normal component of fluid velocity zero on the boundary. Plane 
quadilateral source elements were used to approximate the structure surface, and the integral equation for the source density 
is replaced by a set of linear algebraic equations for the values of 
the source density on the quadrilateral elements. By solving this 
set of equations, the flow velocity both on and off the surface was 
calculated. Besides, Wu et al. also studied on the motion of a 
moored semi-submersible in regular waves and wave induced 
internal forces numerically and experimentally [4]. In their 
mathematical formulation, the moored semi-submersible was 
modelled as an externally constrained floating body in waves, and 
derived the linearized equation of motion.

Comparison between the capability of potential theory and 
viscous fluid theory to predict the fluid characteristic in the 
narrow gaps between the floating bodies was studied by Lin Lu 
et.al. Their simulation result showed that the potential theory over 
predicted the fluid resonance amplitude but it can correct by 
modifying the theory with included artificial damping term, \( \mu = 0.4 - 0.5 \) [18].

Yilmaz and Incecik analyzed the excessive motion of 
moored semi-submersible [5]. They developed and employed two 
different time domain techniques due to mooring stiffness, 
viscous drag forces and damping. In the first technique, first-order 
wave forces acting on structure which considered as a solitary 
excitation forces and evaluated according Morison equation. In 
second technique, they used mean drift forces to calculate slowly 
varying wave forces and simulate for slow varying and steady 
motions. Söylemez developed a technique to predict damaged 
semi-submersible motion under wind, current and wave [6]. He 
used Newton’s second law for approaching equation of motion 
and developed numerical technique of nonlinear equations for 
intact and damaged condition in time domain.

Clauss et al. analyzed the sea-keeping behavior of a semi-
submersible in rough waves in the North Sea numerically and 
experimentally [7]. They used panel method TiMIT (Time-
domain investigations, developed at the Massachusetts Institute of 
Technology) for wave/structure interactions in time domain. The 
theory behind TiMIT is strictly linear and thus applicable for 
moderate sea condition only.

An important requirement for a unit with drilling capabilities 
is the low level of motions in the vertical plane motions induced 
by heave, roll and pitch. Matos et al. were investigated second-
order resonant of a deep-draft semi-submersible heave, roll and 
pitch motions numerically and experimentally [8]. One of the 
manners to improve the hydrodynamic behavior of a semi-
submersible is to increase the draft. The low frequency forces 
computation has been performed in the frequency domain by 
WAMIT a commercial Boundary Element Method (BEM) code. They generated different number of mesh on the structure and 
calculated pitch forces.

Due to the complexity of actual structures’ hull form, S. 
Nallayarasu and P. Siva Prasad were used experimental and 
numerical software (ANSYS AQWA) to study the hydrodynamic 
response of an offshore spar structure which linked to semi-
submersible under regular waves. From both the experimental and 
numerical result, it is obtained that the response of the spar is 
reduced after linked to semi-submersible due to the interaction of 
radiation wave generated by both the structures and the motion of 
spar may be reduced by semi-submersible. However, the research 
also obtained that the motion response for unmoored semi-
submersible is increased when linked to spar [19].

Wackers et al. was reviewed the surface descretisation 
methods for CFD application with different code [9]. Besides, 
simulation of fluid flow Characteristic around Rounded-Shape 
FPSO was also conducted by A. Efi et al. using RANS Method 
[10]. Jaswar et al. were also developed integrated CFD simulation 
software to analyze hull performance of VLCC tanker. The 
integrated CFD simulation tool was developed based on potential 
ty theory and able to simulate wave profile, wave resistance and 
pressure distribution around ship hull [11].

In addition, few experiment tests were carried out to obtain 
the motion response of semi-submersible. A model test related to 
interaction between semi-submersible and TLP was carried out by 
Hassan Abyn et al. [12]. In continue Hassan Abyn et al. also tried 
to simulate the motion of semi-submersible by using HydroSTAR 
and then analyse the effect of meshing number to the accuracy 
of execution result and execution time [13]. Besides, C. L. Siow et 
al. also make a comparison on the motion of semi-submersible 
when it alone to interaction condition by experimental approach 
[14]. Besides that, K.U. Tiau (2012) was simulating the motion of 
mobile floating harbour which has similar hull form as semi-

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submersible by using Morison Equation [15].

### 3.0 MATHEMATICAL MODEL

#### 3.1 Diffraction Potential

In this study, diffraction potential theory was used to obtain the wave force act on the semi-submersible structure, added mass and damping for all six directions of motions. The regular wave acting on floating bodies can be described by velocity potential. The velocity potential normally written in respective to the flow direction and time as below:

\[
\Phi(x, y, z) = Re[\phi(x, y, z)e^{i\omega t}]
\]  

\[
\phi(x, y, z) = \frac{g}{2w} \left( \phi_0(x, y, z) + \phi_s(x, y, z) \right) + \sum_{j=1}^{6} iwX_j \phi_j(x, y, z)
\]  

where; \(g\) is gravity acceleration, \(c_d\) is incident wave amplitude, \(X_j\) is motions amplitude, \(\phi_0\), \(\phi_s\) and \(\phi_j\) are incident wave potential, scattering wave potential, and radiation wave potential due to motions and \(j\) is direction of motion.

From the above equation, it is shown that total wave potential in the system is contributed by the potential of the incident wave, scattering wave and radiation wave. In addition, the phase and amplitude for both the incident wave and scattering wave is assumed to be the same. However, radiation wave potentials are affected by each type of motion of floating body inside system, where the total potential for radiation wave for the body is the summation of the radiation wave generates by each type of body motion such as roll, pitch, yaw, surge, sway and heave.

The diffraction wave potential \(\phi^{(d)}\) must be satisfied with boundary conditions as below:

\[
\nabla^2 \phi^{(d)} = 0 \quad \text{for} \quad 0 \leq z \leq h
\]  

\[
\frac{\partial \phi^{(d)}}{\partial z} + k \phi^{(d)} = 0 \quad \left( k = \frac{w}{g} \right)
\]  

\[
\frac{\partial \phi^{(d)}}{\partial z} = 0 \quad \text{at} \quad z = h
\]  

\[
\phi^{(d)} \sim \frac{1}{r^3} e^{-ik_0z} \quad \text{should be 0 if} \quad r \rightarrow \infty
\]  

\[
\frac{\partial \phi^{(d)}}{\partial n} = \frac{\partial \phi_0}{\partial n} \text{on the body boundary}
\]  

#### 3.2 Wave Potential

By considering the wave potential only affected by structure’s surface, \(S_0\), the wave potential at any point can be presented by following equation:

\[
\phi(P) = \iint_{S_0} \left( \frac{\partial \phi}{\partial \eta} G(P, Q) - \phi(Q) \frac{\partial G(P, Q)}{\partial \eta} \right) dS(Q)
\]  

where \(P = (x, y, z)\) represents fluid flow pointed at any coordinate and \(Q = (\xi, \eta, \zeta)\) represent any coordinate, \((x, y, z)\) on structure surface, \(S_0\). The green function can be applied here to estimate the strength of the wave flow potential. The green function in eq. (8) can be summarized as follow:

\[
G(P; Q) = -\frac{1}{4\pi} \frac{1}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2 + H(x - \xi, y - \eta, z + \zeta)}}
\]  

where; \(H(x - \xi, y - \eta, z + \zeta)\) in eq. (9) represent the effect of free surface and can be solved by second kind of Bessel function.

### 3.3 Wave Force, Added Mass and Damping

The wave force or moment acts on the structure to cause the motions of structure can be obtained by integral the diffraction wave potential along the structure surface.

\[
E_i = -\iint_{S_0} \phi_d(x, y, z) n_i dS
\]  

Also, the added mass, \(A_{ij}\) and damping, \(B_{ij}\) for each motion can be obtained by integral the radiation wave due to each motion along the structure surface.

\[
A_{ij} = -\rho \iint_{S_0} Re \left[ \phi_j(x, y, z) \right] n_i dS
\]  

\[
B_{ij} = -\rho w m \left[ \phi_j(x, y, z) \right] n_i dS
\]  

\[
\eta_i \text{in eq. (10) to eq. (12) is the normal vector for each direction of motion}, \quad i = 1~6 \text{ represent the direction of motion and} \quad j = 1~6 \text{ represent the six type of motions}
\]  

#### 3.4 Viscous Damping

The modified viscous damping from the equation provided by S. Nallayarasu and P. Siva Prasad [19] is shown as follows expression:

\[
b_v = \nu \left( [M + A_{33}] w_1 \right) C
\]  

Where \(b_v\) is heave viscous damping of the floating structure, \(\nu\) is damping ratio for heave, \(M\) is the mass of the floating structure, \(A_{33}\) is heave added mass of floating structure and it is calculated from diffraction potential theory and \(w_1\) is heave natural frequency and \(C\) is the constant for the viscous damping.

The damping ratio, \(\nu\) and heave natural frequency, \(\omega\) at the equation (13) can be found from heave decay experiment. Based on the result obtained from heave decay experiment, the logarithmic decrement method which defines the natural log of the amplitude of any two peaks can be used to find the damping ratio of an under-damped system. The equation for the logarithmic decrement, \(\delta\) as follows

\[
\delta = \frac{1}{n} \ln \left( \frac{x_0}{x_n} \right)
\]  

where \(x_0\) is the first peak amplitude and \(x_n\) is the n-th peak amplitude. After the logarithmic decrement, \(\delta\) found, the damping ratio can be found from the following equation:
Besides, the heave decay experiment also can be used to obtain the damped natural frequency, \( w_d \) and heave natural frequency, \( w_n \) by following equation:

\[
v = \frac{\delta}{\sqrt{2g \cdot 4\pi^2}}
\]

(15)

where the variable \( T \) is period of heave oscillation motion or time required for two continue successive amplitude peaks.

By insert the data obtained from heave decay experiment into equation (13) the heave viscous damping will able to calculate and insert into the motion equation follow:

\[
(M + A_{33})\ddot{x} + (B_{33} + h_v)\dot{x} + c\dot{x} + F = 0
\]

(18)

where the \( M \) is structure mass, \( A_{33} \) is heave added mass, \( B_{33} \) is linear damping from diffraction potential theory, \( h_v \) is the viscous damping defined at equation (13), \( c \) is the heave restoring force, and \( F \) is the wave force contributed to heave motion.

3.0 MODEL PARTICULAR

As mentioned, the semi-submersible model was selected as the test model in this study. This Semi-submersible model was constructed based on GVA 4000. The model is constructed from four circular columns connected to two pontoons and two braces. Two pieces of plywood are fastened to the top of the Semi-submersible to act as two decks to mount the test instruments. The model was constructed from wood following the scale of 1:70 (Table 1).

Upon the model complete constructed, few tests were carried out to obtain the model particulars. Inclining test, swing frame test, oscillating test, decay test and bifilar test were carried out to identify the hydrostatic particular for the semi-submersible. The dimension and measured data for the model was summarized as in table 1.

<table>
<thead>
<tr>
<th>Table 1 Principal particular of the Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>0.954 m</td>
</tr>
</tbody>
</table>

4.0 MESHING FOR SEMI-SUBMERSIBLE

In this study, the numerical method applies to executing the motion response of semi-submersible will only estimate the wave force acting on the surface of the port side structure of semi-submersible. After that, the total wave force for the semi-submersible is double before it fixed into the motion equation.

The selected semi-submersible model in this study is constructed based on GVA 4000 type. Total panels used in the execution are 272 where 25 panels on each column and 222 panels on pontoon surface. The example meshing constructed by this numerical method for the semi-submersible model is shown in figure 1.

5.0 RESULT

5.1 Heave Decay Experiment

The heave decay experiment was carried out to collect the required data, such as damping ratio and natural heave frequency to able the programming to execute the heave viscous damping. This decay experiment was carried out by displacing the model in the heave directions or along the heave axes, releasing and recording the displacement time histories. The tests are repeated when necessary to obtain reliable results. The data collected from decay experiment after that was processed to obtain the required information for the execution of heave viscous damping follow the discussion in part 3.4. From the calculation, it is obtained that the heave damping ratio, \( \nu \) is 1.628 %, while the heave natural frequency is 3.23 rad/Sec in model scale. The time domain heave decay data collected from the experiment is shown in figure 2.

5.1 Heave Motion Response

In this part, the heave response amplitude for GVA 4000 semi-submersible structure in head sea condition was discussed. The result from the proposed numerical result was also compared to the motion experiment result. Input for the numerical program was also adjusted to make the condition as close as the experimental condition. Since this paper is targeted to discuss the involving of viscous damping in calculation to correct the heave motion.
motion predicted by the diffraction potential theory, thus the discussion will only made on the heave motion.

From the previous study, it can be obtained that the diffraction potential theory is weak in predicting the heave motion response when the wave frequency closes to the structure natural frequency. By detail study of the problem, it is obtained that the linear damping predicted by the diffraction potential theory is very small at this wave condition. This error was caused the theory to give the infinite response at the wave condition where it closer to semi-submersible natural frequency.

The figure 3 shown the non-dimensional heave damping calculated by diffraction potential theory, proposed viscous damping equation and the summation of both the heave damping. From the comparison, it is obtained that the heave damping calculated by diffraction potential theory increase rapidly when the wave length increase if the wavelength is below 1.5 meters. If the wavelength longer than 1.5 meters, it is obtained that the damping coefficient calculated by diffraction potential theory decrease significantly by increase the wavelength. The linear damping from diffraction potential theory finally reduced to nearly zero after wave length 5 meters. From the study, the region where the heave motion dominates by damping term is located in the structure natural frequency region where this region is located at wavelength around 9 meters for this selected semi-submersible model. By referring to the figure 3, the damping coefficient obtained by summing up the damping calculated by both the methods given the magnitude around 0.275. This damping magnitude was helped to avoid the motion equation to divide by zero damping then trend the motion response to infinity. At this calculated magnitude, it can be summarized that the additional viscous damping by using proposed equation can be helped to correct the damping coefficient and then corrected the motion response estimated at the damping dominated region.

On the other hand, the heave viscous damping calculated by proposing equation is increasing gradually when the wave length increased. This additional heave viscous damping added into the motion equation will help to increase the damping coefficient and to avoid the large overshooting of the heave motion response when wave frequency is close to semi-submersible’s heaved natural frequency.

To obtain the total damping for semi-submersible heave motion, the magnitude of damping coefficient is assumed can be directly sum up for the damping calculated by both the methods. As shown in figure 4, total damping by sum-up the executed damping between the two methods will be influenced by the damping coefficient calculated by two of the methods for the wavelength below 5 meters. However, due to the damping calculated by diffraction potential theory for this semi-submersible is trending to become zero for wavelength longer than 5 meters, then the tendency of total damping is following the tendency of the viscous damping calculated by the proposed equation. At understood the heave motion dominated by damping term is located in the structure natural frequency region where this region is located at wavelength around 9 meters for this selected semi-submersible model. By referring to the figure 3, the damping coefficient obtained by summing up the damping calculated by both the methods given the magnitude around 0.275. This damping magnitude was helped to avoid the motion equation to divide by zero damping then trend the motion response to infinity. At this calculated magnitude, it can be summarized that the additional viscous damping by using proposed equation can be helped to correct the damping coefficient and then corrected the motion response estimated at the damping dominated region.

The heave RAO calculated by the diffraction potential theory and the corrected diffraction potential theory by viscous damping correction is presented in figure 4. The experimental data collected is only ranged from wavelength 1 meter to wavelength around 9 meters due to the limitation of the wave generating device in the laboratory. Comparison between the heave response calculated by diffraction potential theory with and without viscous damping correction is presented in the figure 4. From the figure, it can be obtained that the diffraction potential theory with
viscous damping correction was reduced the infinity heave response to the reasonable range compared to the result obtained by diffraction potential theory without viscous damping correction.

The tendency of the heave response calculated by the diffraction potential theory with and without viscous damping correction is similar between each other. However, due to the extra damping effect of the viscous damping correction, the heave response is no overshoot to infinity compared to the results obtained from pure diffraction potential theory at wave frequency closed for the structure natural frequency (wave length equal to 9 meters). This observation also shown that a good prediction of viscous damping is significantly important to estimate the heave motion response for semi-submersible structure at damping dominated region. Therefore, it can be summarized that the neglected of the viscous effect on the estimate heave response of semi-submersible like the diffraction potential theory will lead to over prediction of heave response at the region where the motion is dominated by damping. The reason for this observation is also explained in the figure 3 where it can obtain that the damping coefficient under-predicted by the diffraction potential theory and it magnitude is almost zero at the wave condition.

![Figure 4: Heave motion response for semi-submersible model](image)

**5.0 Conclusion**

In the conclusion, this paper was presented the viscous damping correction method to improve the tendency of heave response calculated by the diffraction potential theory. In general, the diffraction potential theory is a good method to predict the motion response of offshore structure especially the semi-submersible structure. In comparison to the experimental result, it is obtained that the pure diffraction potential theory is weak in predicting the heave response in the region where the heave motion is dominated by damping or drag term. The weakness of the diffraction potential theory to neglect the viscous effect was caused the damping smaller and lead to wrong heave response tendency at the damping dominated region.

By involving the viscous damping correction using the proposed equation, the small damping magnitude calculated from diffraction potential theory can be corrected. In this paper, the numerical results calculated by the proposed method shown that the overshooting problem observed at pure diffraction potential theory had improved. At the same time, accuracy of heave motion response in head sea condition calculated by the viscous damping correction method is still agreed to experiment result. Therefore, it can be concluded that the viscous damping correction is required to consider whether the accuracy of the estimated heave motion of offshore floating structure such as semi-submersible is important.

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**REFERENCE**


