

# 3D Independent Actuators Control Based On A Proportional Derivative Active Force Control

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## ABSTRACT

This paper presents a control method for 3D independent actuators using PDAFC (Proportional Derivative Active Force Control). PD is used to stabilize the actuators, where as AFC is used to reject disturbance uncertainty by estimating disturbance torque value of actuator. Simulation result shows that PDAFC can minimize disturbance uncertainty effect. To test the performances of a control system actuator given disorder constant and sine. To research it uses cube as an actuator. when give the actuators disturbance with the constant value of 45, to achieve setpoint actuators takes 1 second. This condition is in all actuators whether it was the yaw, pitch and roll. When give the actuators disturbance with the sine, to achieve setpoint actuators takes 1.25 second. This condition is in all actuators whether it was the yaw, pitch and roll.

**KEY WORDS:** *Inertia, Reaction Wheel, PID, AFC*

## NOMENCLATURE

AFC	Active Force Control
PD	Proportional Derivative
PID	Proportional Integral Derivative
PDAFC	Proportional Derivative Active Force Control
LQR	Linear Quadratic Regulator
TTE	Trajectory Tracking Error
Q	Disturbance

## 1.0 INTRODUCTION

This paper presents a method of control actuators independent 3D. On this experiment as an actuator using cube mounted reaction wheel, as in a figure 1. Reaction wheel on the cube is used to produce the moment of inertia so that the cube can be controlled. In this research control algorithm used is PDAFC (Proportional Derivative Active Force Control). There are many research pertaining to the reaction wheel as actuator with a variety of control algorithms.

Mohanarajah. from the Swiss Federal Institute of Technology controls cube using algorithms LQR (Linear Quadratic Regulator) for 2D and 3D actuator [1,2].

Snider from the Air Force Institute of Technology, USA conducting research controlling attitude on the satellite using reaction wheel and Control algorithms used is PID. Controlled system has a change in attitude at high speed, by using reaction wheel a system can be controlled properly [3].

Shirazi from Toosi University of Technology, Iran Conducting research draws up reaction wheel shaped like a pyramid, used to evaluate the performance of the satellite attitude control, with the goal of minimizing the total consumption of power from the system [4].

Pranajaya from University of Toronto, Canada researching on the nano a satellite with the orbit of low to detect and identify a signal transmitted by a ship (Automatic Identification System). This satellite known as nano satellites Ship Tracking [5].

Mayer from Osaka University, Japan conduct the research uses a reaction wheel as an actuator to influence the movement of the robot in two ways: (1) The roll and yaw stabilized by the rotation of the rotor, the higher the speed of the rotor then slows the robot to react to disturbances. (2) His area in stabilizing by the acceleration of the rotor control using the principle of reaction the act [6].

Pitowarno from the Electronics Engineering Polytechnic Institute of Surabaya had designed Active Force Control and Knowledge-Based System for planar two-joint robot arm to improve performance of Active Force Control [7].

Ajorkar from the Amirkabir University of Technology, Tehran Conducting research to develop adaptive algorithms, neural network control of the systems. Actuators are used for satellite attitude control using 4 configuration reaction wheels [8].

Muehlebach from the Swiss Federal Institute of Technology nonlinear analysis and control of a reaction wheel based 3d invert pendulum [9].

To control actuator needed algorithms control, the more simple an algorithm is getting better. With the control algorithm, actuator stability is controlled despite getting disturbingly from the environment. The more complicated algorithms need a computing performance high. On this research control PD (Proportional Derivative) serves to stabilize actuator, but it still is not enough if there is any disturbance of the environment. Therefore, added the AFC (Active Force Control) control has the ability to cancel the disturbances that occurs on the actuator.

The purpose of this research is controlling actuator 3D independently use algorithms control PD and AFC. This paper compiled divided into several chapters. Chapter 1 introduction, chapter 2 presenting modeling a system actuator, chapter 3 actuator controller designs. Chapter 4 provides the performance of the controller shown in the simulation numerical, and chapter 5 conclusion of the research.

## 2.0 ACTUATOR MODELING

Before designing the controller, in this section the mathematical model of the actuator will be presented :

### 2.1. Kinematic Equation

Consider the Cube balancing on its corner, as shown in Figure. 1. Let  $\omega_h \in \mathbb{R}^3$  denote the angular velocity of the cube relative to the inertial frame  $\{I\}$  expressed in the Cube's body fixed frame  $\{B\}$ ,  $\omega_{w_i} \in \mathbb{R}^3$ ,  $i=1,2,3$  the angular velocities of the wheels around the rotational axis  $e^{B_i}$  and  ${}^I_R \in SO(3)$  denote the attitude of the Cube relative to the inertial frame  $\{I\}$ . Note that attitude of the Cube can also be represented by the xyz-angle Euler (Yaw ( $\alpha$ ), Pitch ( $\beta$ ), and Roll ( $\gamma$ )). The relationship between the rotation matrix and Euler angle representation is given by :

$${}^I_R = e^{\tilde{e}^3 \alpha} e^{\tilde{e}^2 \beta} e^{\tilde{e}^1 \gamma} \quad (1)$$

where  $e$  is the matrix exponential. The nonlinear system dynamics are derived using Kane's equation for multi bodies given by :

$$\delta_v^T \sum_{j \in \mathcal{B}} [J_{p_j}^T \dot{p}_j + J_{R_j}^T \dot{R}_j - J_{p_j}^T F_j^a - J_{R_j}^T T_j^a] = 0 \quad (2)$$

$$v := (\omega_h, \omega_{w_1}, \omega_{w_2}, \omega_{w_3}) \in \mathbb{R}^6 \quad (3)$$

denotes generalized velocity,  $j$  denotes a particular rigid body of the multi-body system,  $p_j$  the linear momentum of the rigid body,  $n_j$  the angular momentum,  $F_j^a$  the external active force,  $T_j^a$  the external active torque, and  $J_{\cdot j}$  the Jacobian matrix. Geometrically, Kane's equation can be interpreted as the projection of the Newton-Euler equation on to the configuration manifold's tangent space.

Consider the Cube as a multi-body system consisting of four

rigid bodies: The Cube housing  $h$  and three reaction wheels  $W1$  (yaw) as in a figure 2,  $W2$  (pitch) as in a figure 3 and  $W3$  (roll) as in a figure 4. consider the Cube housing  $h$  and let  $h$  and let  $\gamma_h$  denote the position of the center of mass of the Cube frame expressed in Cube body fixed frame  $\{B\}$  now, using

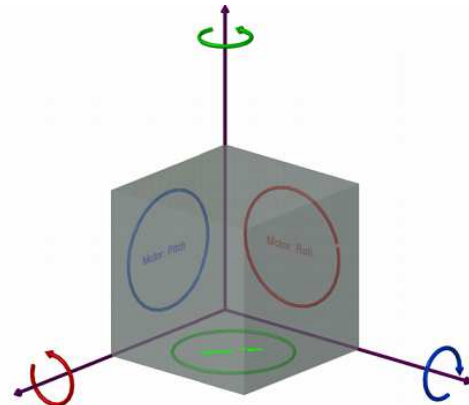


Figure 1: Cube Actuator

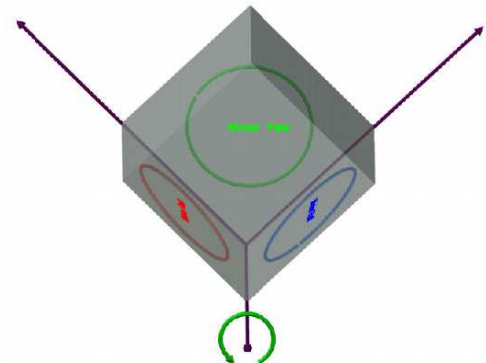


Figure 2: Actuator with the yaw rotation

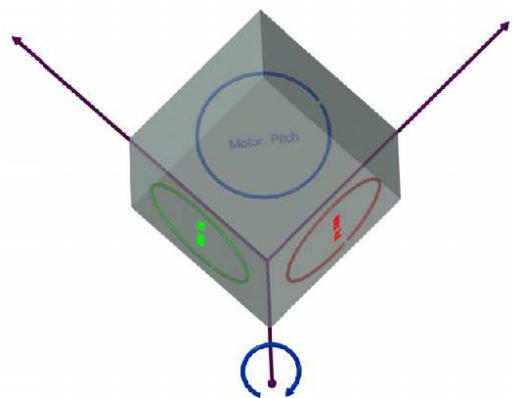


Figure 3: Actuator with the pitch rotation

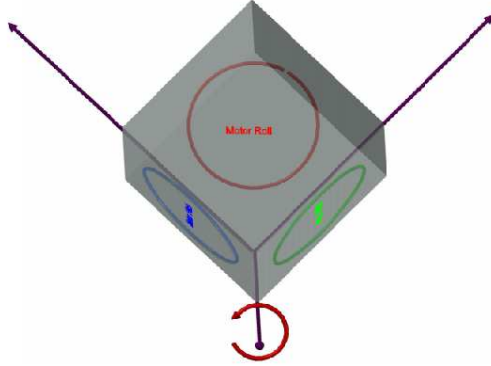


Figure 4: Actuator with the roll rotation

$$\dot{Y}_h = \omega_h x Y_h \quad (4)$$

$$\dot{Y}_h = \dot{\omega}_h x Y_h + \omega_h x (\omega_h x Y_h) \quad (5)$$

The time derivative of the housing's linear momentum is given by

$$\dot{p}_h m_h (\dot{\omega}_h x Y_h + \omega_h x (\omega_h x Y_h)) \quad (6)$$

Similarly, the time derivative of the housing's angular momentum  $n_h = \Theta_h \omega_h$  is given by

$$\dot{n}_h = \Theta_h \dot{\omega}_h - (\Theta_h \omega_h) x \omega_h \quad (7)$$

where  $\Theta_h \in \mathbb{R}^{3 \times 3}$  is the inertia tensor of the Cube housing h Next, consider the  $i^{\text{th}}$  wheel  $i=1,2,3$  with angular velocity given by  $\omega_{wi} e_i + \omega_h$ . The time derivative of the wheel's linear momentum is given by

$$\dot{p}_{wi} = m_w (\dot{\omega}_h x Y_{wi} + \omega_h x (\omega_h x Y_{wi})) \quad (8)$$

Where  $Y_{wi}$  the position of wheel center of mass. The angular momentum of the wheel and its time derivative are given by

$$n_{wi} = \Theta_{wi} \omega_h + \Theta_{wi} (i, i) \omega_{wi} e_i \quad (9)$$

Where  $\Theta_{wi} \in \mathbb{R}^{3 \times 3}$  is the inertia tensor of the reaction wheel  $w_i$ . The Jacobian matrices related to the Cube housing h are given by

$$J_{P_h} = \frac{\partial}{\partial v} \dot{Y}_h = (-\dot{Y}_h, 0, 0, 0) \in \mathbb{R}^{3 \times 6} \quad (10)$$

$$J_{R_h} = \frac{\partial}{\partial v} \dot{\omega}_h = (I, 0, 0, 0) \in \mathbb{R}^{3 \times 6} \quad (11)$$

and Jacobian matrices related to the wheels are given by

$$J_{P_{wi}} = \frac{\partial}{\partial v} \dot{Y}_{wi} = (-\dot{Y}_{wi}, 0, 0, 0) \in \mathbb{R}^{3 \times 6} \quad (12)$$

$$J_{R_{wi}} = \frac{\partial}{\partial v} (\omega_{wi} e_i + \omega_h) = (I, \delta_i) \in \mathbb{R}^{3 \times 6} \quad (13)$$

Where  $\delta_i \in \mathbb{R}^{3 \times 3}$  has all zero elements except for the  $i^{\text{th}}$  diagonal element, which is one. The active torque on the Cube housing and the wheels are given by

$$\begin{aligned} -T_h &= (T_{w1}, T_{w2}, T_{w3}) \\ &= K_m u - C_w (\omega_{w1}, \omega_{w2}, \omega_{w3}) \end{aligned} \quad (14)$$

Where  $K_m$  motor constant  $u := (u_1, u_2, u_3) \in \mathbb{R}^3$  is the current input of each motor driving the wheels, and  $C_w$  is the damping constant. Finally, gravity  $g$  leads to an active force on all bodies. Note that,  $g$  is expressed in the body fixed frame  $\{B\}$  and given by

$$g(\mathcal{O}) = g_0 (s(\beta), -s(\gamma)c(\beta), -c(\gamma)c(\beta)) \quad (15)$$

Where  $g_0 = 9.82 m.s^{-2}$

Now, inserting (24) to (32) into (27) the following equations of motion

$$\begin{aligned} \hat{\Theta} \dot{\omega}_h &= \Theta \omega_h x \omega_h + Mg + \Theta_w \omega_w x \omega_h - (K_m u - C_w \omega_w) \\ \Theta \dot{\omega}_h &= K_m u - C_w \omega_w - \Theta_w \dot{\omega}_h, i=1,2,3 \end{aligned} \quad (16)$$

$$M = m_h \tilde{Y}_h + \sum_{i=1}^3 m_{wi} \tilde{Y}_{wi}$$

$$\Theta = \Theta_h - m_h \tilde{Y}_h^2 + \sum_{i=1}^3 \left[ \Theta_{wi} - m_{wi} \tilde{Y}_{wi}^2 \right]$$

$$\Theta_w = \text{diag}(\Theta_{w1}(1,1), \Theta_{w2}(2,2), \Theta_{w3}(3,3))$$

$$\hat{\Theta} = \Theta - \Theta_w$$

Finally, the kinematic equation of the Cube is given by [1]

$$\omega_h = \begin{pmatrix} -s(\beta) & 0 & 1 \\ s(\gamma)c(\beta) & c(\gamma) & 0 \\ c(\gamma)c(\beta) & -s(\gamma) & 0 \end{pmatrix} \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} \quad (17)$$

## 2.2. Dynamic Equation

Let  $\varphi$  and  $\psi$  describe the positions of the 1D inverted pendulum as shown in Figure 5. Next, let  $\Theta_w$  denote the reaction wheel's moment of inertia,  $\Theta_0$  denote the system's total moment of inertia around the pivot point in the body fixed coordinate frame, and  $m_{tot}$  and  $l$  represent the total mass and distance between the pivot point to the center of gravity of the whole system.

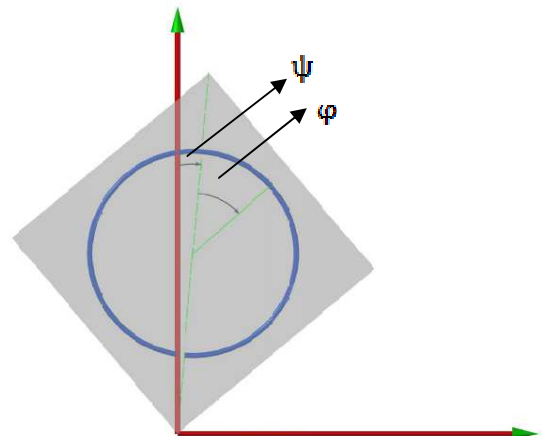


Figure 5: Reaction wheel based 1D

The Lagrangian of the system is given by

$$L = \frac{1}{2} \dot{\theta}_0 \dot{\phi}^2 + \frac{1}{2} \dot{\theta}_w (\dot{\phi} + \dot{\psi})^2 - mg \cos \phi \quad (18)$$

where  $\dot{\theta}_0 = \dot{\theta}_0 - \dot{\theta}_w > 0$   $m = m_{tot}$  and  $g$  is the constant gravitational acceleration. The generalized moment are defined by

$$p_\phi := \frac{\partial L}{\partial \dot{\phi}} = \dot{\theta}_0 \dot{\phi} + \dot{\theta}_w \dot{\psi} \quad (19)$$

$$p_\psi := \frac{\partial L}{\partial \dot{\psi}} = \dot{\theta}_w (\dot{\phi} + \dot{\psi}) \quad (20)$$

Let  $T$  denote the torque applied to the reaction wheel by the motor. Now, the equations of motion can be derived using the Euler-Lagrange equations with the torque  $T$  as a non potential force. This yield

$$\dot{p}_\phi = \frac{\partial L}{\partial \phi} = mg \sin \phi \quad (21)$$

$$\dot{p}_\psi = \frac{\partial L}{\partial \psi} = T \quad (22)$$

Note that the introduction of the generalized moment in (19) and (20) leads to a simplified representation of the system, where (21) resembles an inverted pendulum augmented by an integrator in (22). Since the actual position of the reaction wheel is not of interest, we introduce  $x := (\phi, p_\phi, p_\psi)$  to represent the reduced set of states and describe the dynamics of the mechanical system as follows[10]:

$$\dot{x} = \begin{pmatrix} \dot{\phi} \\ \dot{p}_\phi \\ \dot{p}_\psi \end{pmatrix} = f(x, T) = \begin{pmatrix} \dot{\theta}_0^{-1} (p_\phi - p_\psi) \\ mg \sin \phi \\ T \end{pmatrix} \quad (23)$$

### 3.0 CONTROLLER DESIGN

In this chapter presents the algorithm control for actuator. The purpose is to control the stability of actuator by combining algorithms control the PD and AFC. Figure 3 is block a diagram of the actuator stability control. The controller design is focused to stabilize the actuator toward disturbance. PD controller is used to stabilize actuator and AFC to reject uncertainty disturbance from environment. In this simulation, actuator get constant and fluctuated.

#### 3.1 AFC Control

Figure 6 shows the Active Force Control (AFC) with IN is -

Figure 6: Diagram block PDAFC

mass of the inertia moment of estimator matrix,  $K_m$  is constant of motor torque,  $I_m$  is current of motor,  $\tau$  is actual motor torque,  $\tau_{act}$  is used actual motor torque,  $\tau'$  is measured motor torque,  $Q$  is disturbance, and  $Q'$  is disturbance estimation.  $\tau'$  can be measured by torque sensor, considering (measuring) the actual motor current multiplied with the motor torque constant. With reference to Figure 3 AFC, the simplified dynamic model of the system can be written as

$$\tau_{act} = \tau + Q = I(\theta) [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act} \quad (24)$$

Where  $I(\theta)$  is the mass of the moment of inertia of reaction wheel and  $\theta$  is angle of each reaction wheel,  $[\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act}$  is acceleration angular of body moving. From figure 3, the measurement of  $Q'$  (i.e., an estimate of the disturbance,  $Q$ ) can be obtained such that

$$Q' = \tau' - [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act}' IN' \quad (25)$$

Using the torque sensor or vice versa, if the current sensor is used then the equation becomes.

$$Q' = I_m' K_m - [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act}' IN' \quad (26)$$

Where the superscript  $(\cdot)$  denotes a measured or estimated quantity. The torque  $\tau$  can be measured directly using a torque sensor or indirectly by means of a current sensor. The actual motor torque can be written as

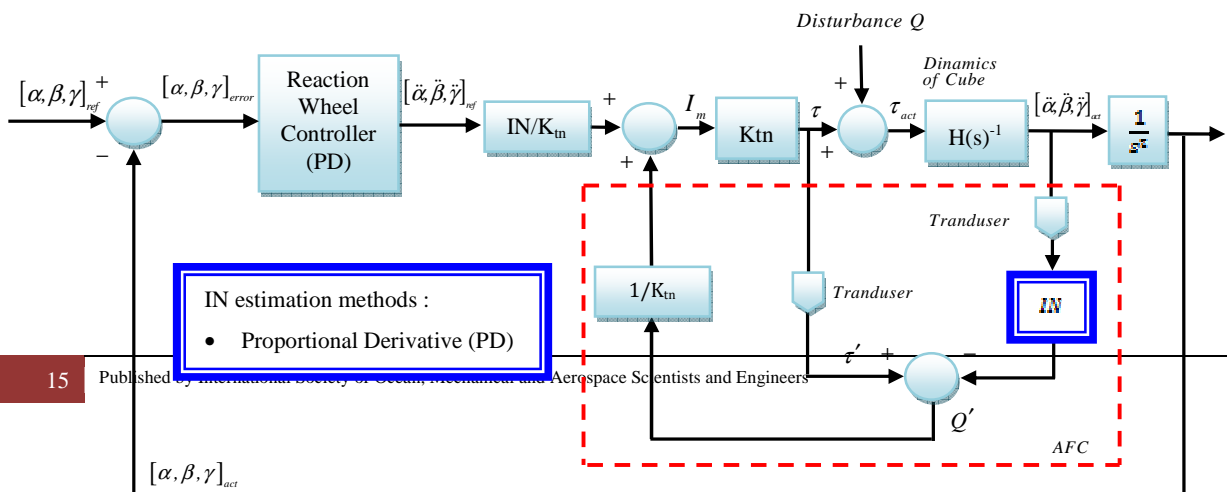
$$\tau = \left( \frac{[\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{ref}' IN' + \frac{Q'}{K_m}}{K_m} \right) K_m \quad (27)$$

Substitute equation (26) into equation (27) and (24), then results a new equation that is.

$$\tau = \left( \frac{[\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{ref}' IN' + \frac{I_m' K_m - [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act}' IN'}{K_m}}{K_m} \right) K_m \quad (28)$$

$$= [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{ref}' IN' + I_m' K_m - [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act}' IN' \quad (29)$$

$$\tau_{act} = IN' \left( [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{ref}' - [\ddot{\alpha} \ddot{\beta} \ddot{\gamma}]_{act}' \right) + I_m' K_m + Q \quad (29)$$



Equation (28) is known as the equation of AFC controller output. It has been confirmed that if the measured values or estimation of the parameters in the equation obtained accurately, it will be a very solid system to reject the disturbances[8].

### 3.2 PDAFC Control

Referring to Figure 3 and equations (28) and (29), where the superscript mark (') indicates a measured or estimated number. Equation (28) shows a proportional form (P) controller in connection with the use of errors acceleration signal. In this context,  $IN'$  can be regarded as a proportional constant. This is the known fact that the proportional controller works without an additional dynamic parameter and has sufficient ability to increase the error steady state from the system because of the system dynamics an uncertainty. An addition of derivative component (D) can enhance the system performance by enforcing the occurred oscillation to the control in a minimum condition.

Assuming that the equation (28) is an acceleration local proportional control and as given that the disturbances are highly nonlinear, varied and unpredictable, modification AFC scheme by incorporating derivative components of the inertia matrix estimator. This is named PDAFC. The equation of derivative controller feedback can be written as follows

$$G_c(t) = [e(t)] IN_p \quad (30)$$

$$G_c(t) = \left[ \frac{de(t)}{dt} \right] IN_d \quad (31)$$

Which of  $G_c$  is control signal, If the error  $e(t)$  is relatively constant  $G_c(t)$  will become large and will hopefully correct the error. By letting  $e(t) = ([\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]_{ref} - [\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]_{act})$  and then incorporating equation (30) into equation (28) to include the additional integral element and then incorporating equation (31) into equation (29) to include the additional derivative element, the proposed algorithm is given by[8]:

$$\tau_{PDAFC} = IN_p ([\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]_{ref} - [\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]_{act}) + IN_d \frac{d([\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]_{ref} - [\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma}]_{act})}{dt} + I_m K_m \quad (32)$$

Which of  $IN_d$  is a derivative constant. Equation (32) is for PDAFC.

## 4.0 SIMULATION RESULT

The simulation test was performed using Simulink to evaluate the performance of the controller. The simulation model was

used in the S - function block. In this simulation the model contains disturbance that has been modeled. PD coefficients that used for simulations were derived by trial and error to get the best performance, the PD parameter is listed in Table 1. The first simulation is done with the PDAFC system without disturbing. The second simulation performed a

PDAFC system with constant disturbance, by setting the value of 45. The third simulation performed PDAFC system with sine disturbance, by setting the value of the frequency of disturbance and the amplitude of the disturbance.

From the attempt to test performance control PDAFC by giving constant disturbance and sine disturbance on the actuators.

Table 1: PD coefficients simulation parameter

Parameter	Value
Kp	5
Ki	7
Kd	0.15
Kpafc	0.02
Kdafc	3
Frequency sine	10 Hz
Amplitude sine	300 Vp-p
Constant	45
Massa cube	2kg
Gravitation	8.9 m/s <sup>2</sup>

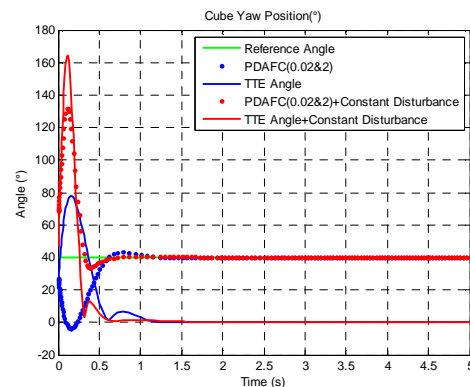


Figure 7 : Yaw Position constant disturbance

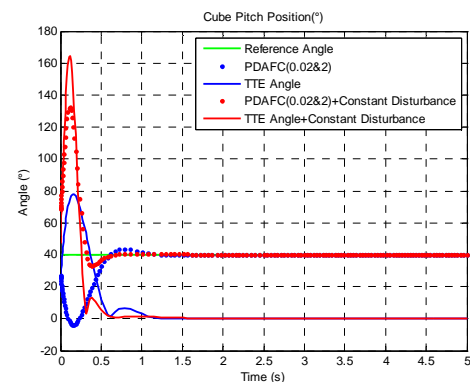


Figure 8: Pitch Position constant disturbance

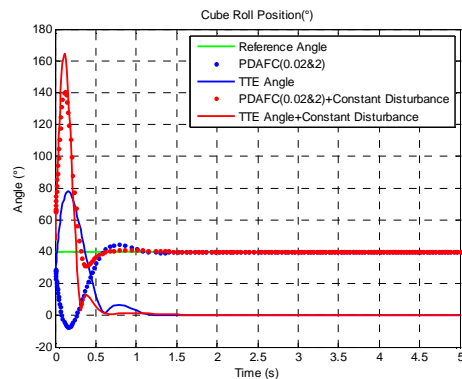


Figure 9: Roll Position constant disturbance

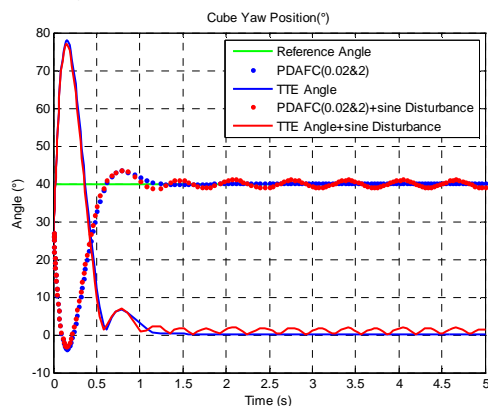


Figure 10: Yaw Position sine disturbance

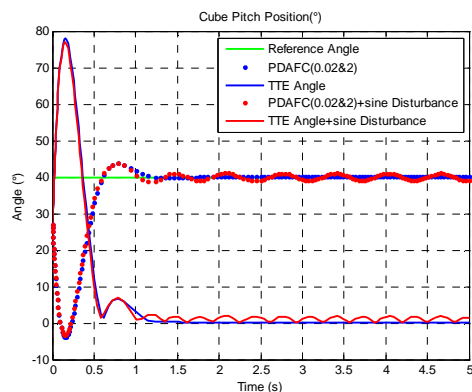


Figure 11: Pitch Position sine disturbance

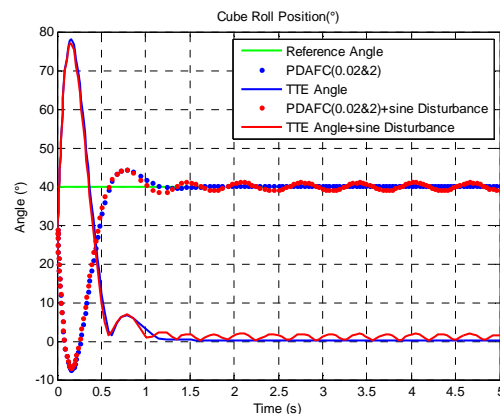


Figure 12: Roll Position sine disturbance

## 5.0 CONCLUSION

The simulation results have been presented to show the performance of the controller. Although there is a disturbance in actuators, actuators can remain stable briefly. 1 seconds to constant disturbance and 1.25 seconds to sine disturbance. If actuator observe time to steady, when actuator with disturbance and without disturbance little the difference. This shows control working very well.

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